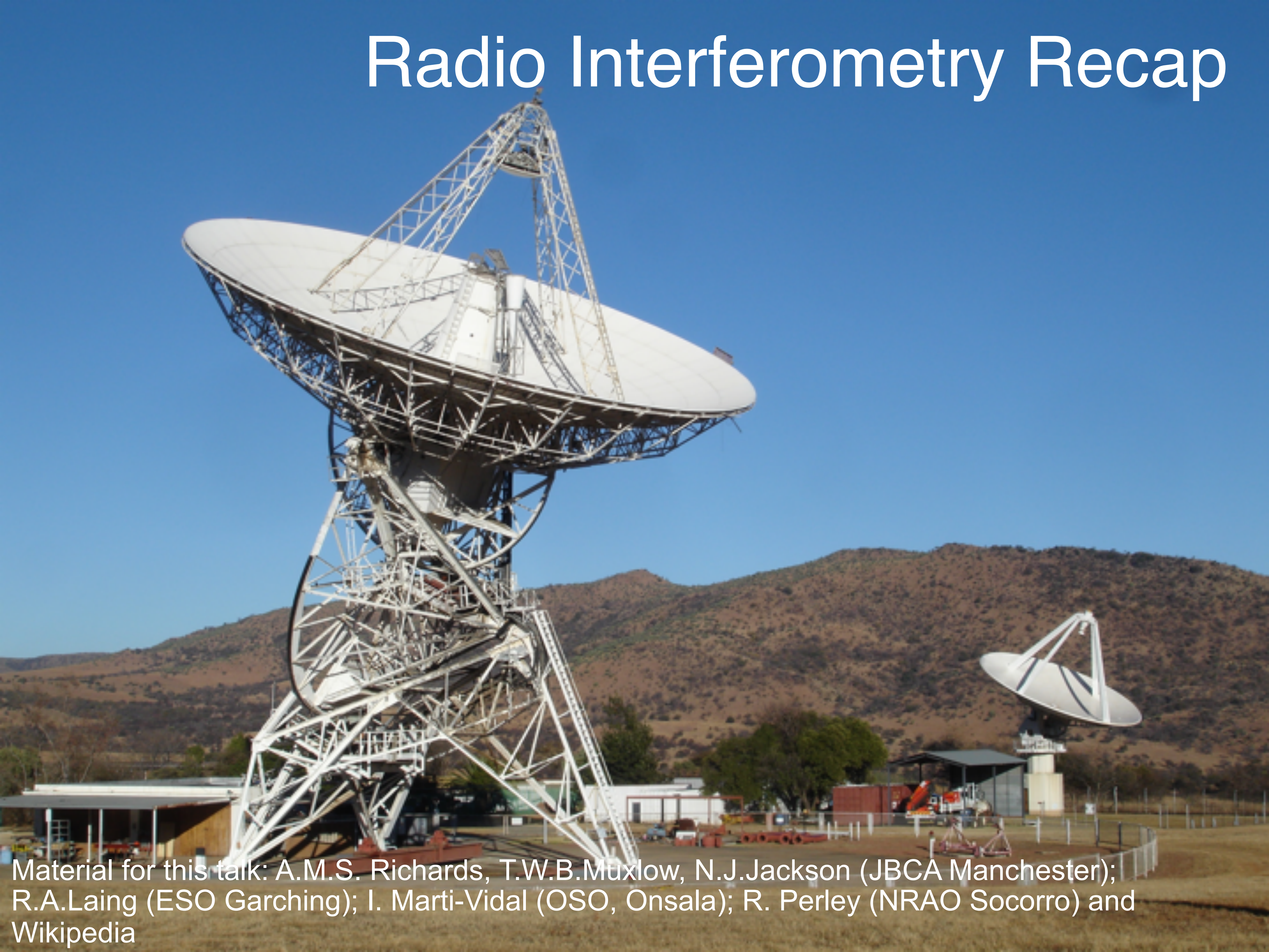
Radio Interferometry Recap

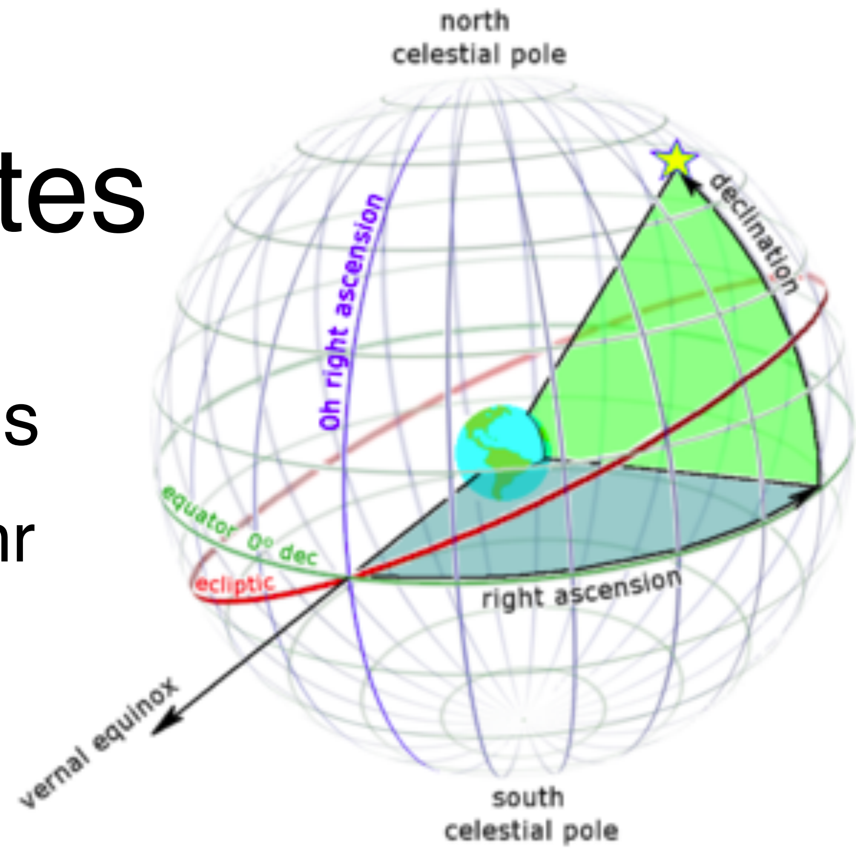


Material for this talk: A.M.S. Richards, T.W.B.Muxlow, N.J.Jackson (JBCA Manchester); R.A.Laing (ESO Garching); I. Marti-Vidal (OSO, Onsala); R. Perley (NRAO Socorro) and Wikipedia

Summary

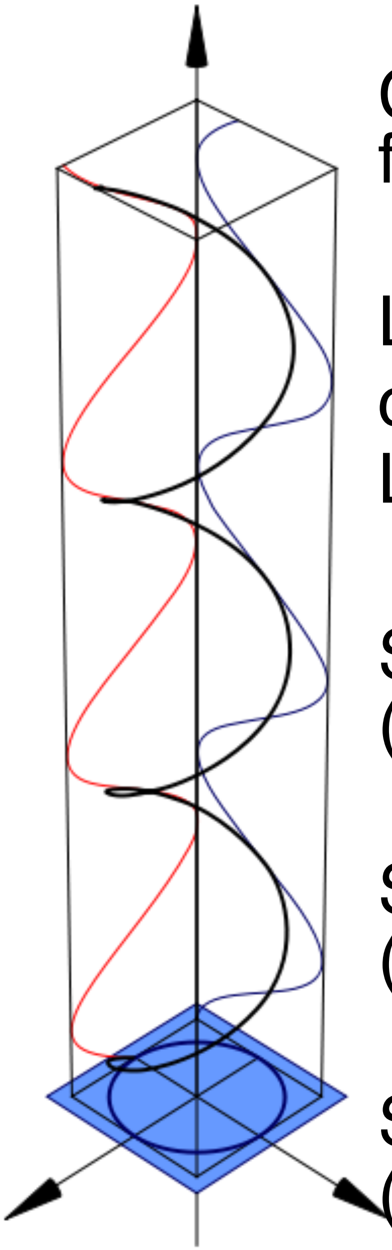
- Radio astronomy – key aspects
- Radio interferometry
  - Earth rotation aperture synthesis
  - Fourier relationship between visibilities & sky brightness
  - Delay and signal path
- Correlation
- Sensitivity of an array
- Image fidelity and missing spacings

Coordinates



- Celestial equatorial coordinates
  - Right Ascension 360deg=24 hr
  - Declination +/-90deg
- (Local) Sidereal Time
  - = Universal Time at spring equinox
  - Advances ~4 min per day with respect to Universal Time
    - (UT ~ Solar time at Greenwich, UK, without summer time!)
- (Local) Hour Angle of celestial object:
  - HA = LST – RA

Polarization jargon: Rx feeds



CIRCULAR
feeds

Left/Right/cross correlations LL RR LR RL

Stokes V = (RR-LL)/2

Stokes Q = (RL + LR)/2

Stokes U = (RL – LR)/2*i*

Diagrams thanks to Wikipedia

LINEAR
feeds

Correlations

XX, YY, XY, YX

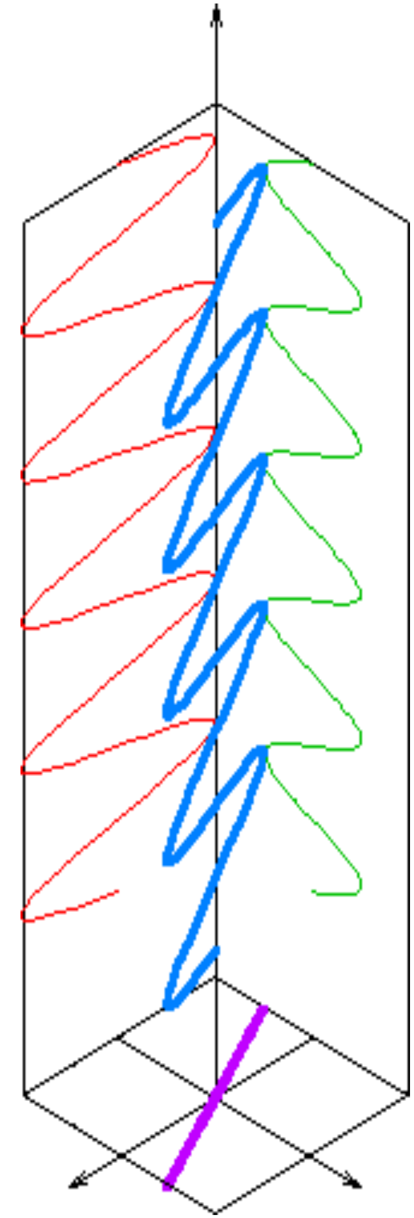
Stokes Q = (XX – YY)/2

Stokes U = (XY – YX)/2

Stokes V = (XY – YX)/2*i*

Polarized intensity P = $\sqrt{(Q^2+U^2+V^2)}$

Polarization angle c = ½ atan(U/Q)



What produces radio waves naturally?

Thermal continuum

Cooler objects peak at longer wavelength

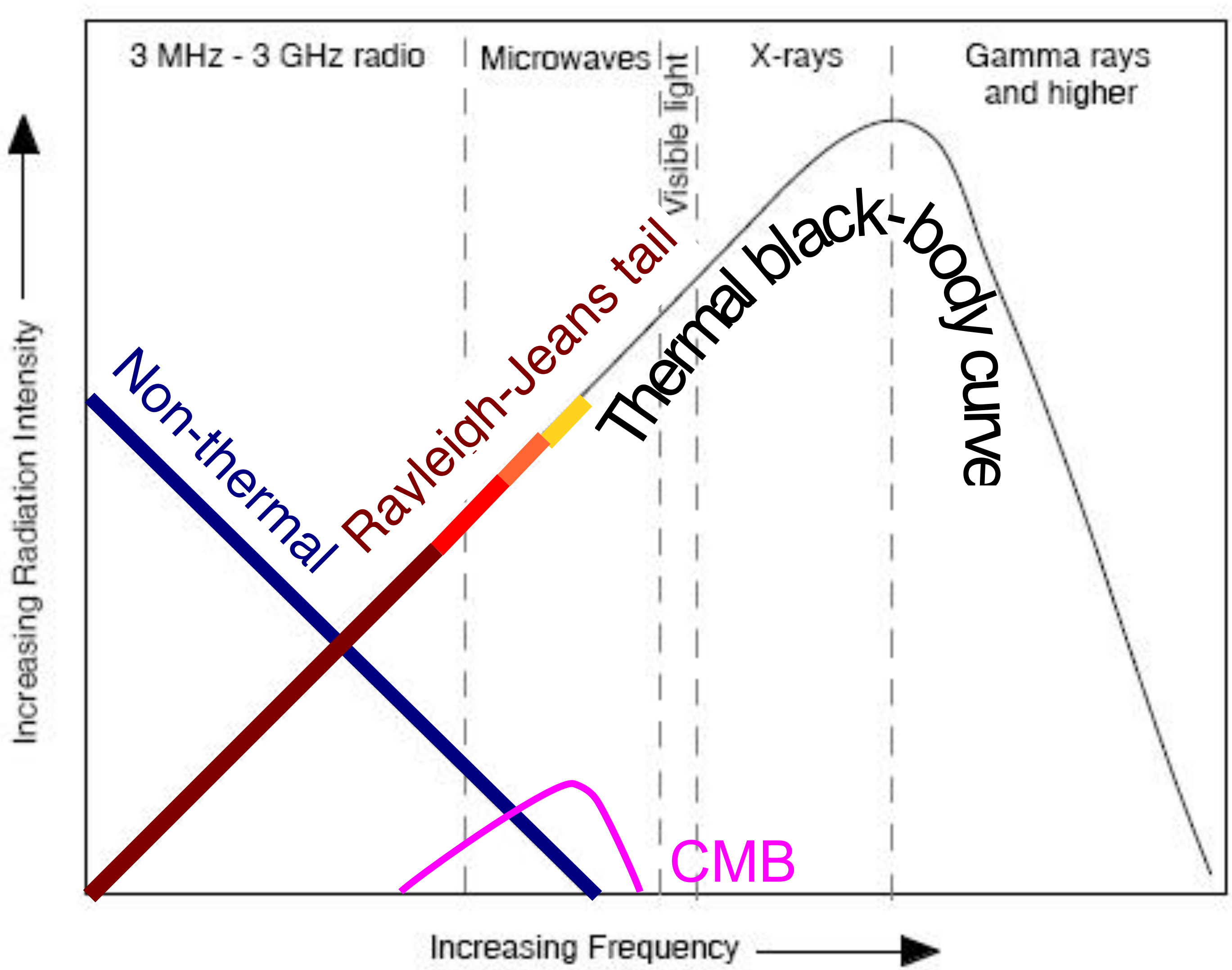
e.g. Stardust; Plasma - stellar atmosphere; Cosmic Microwave Background

Non-thermal emission

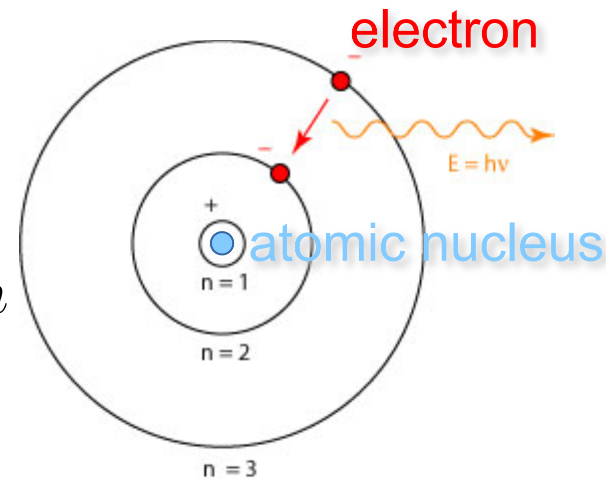
Electrons accelerated in a magnetic field

Brighter at longer wavelength

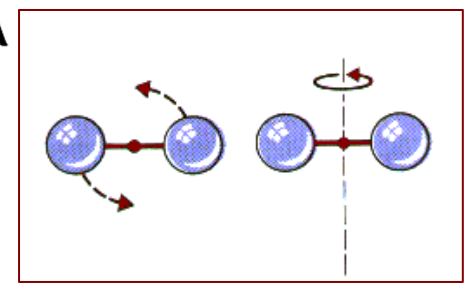
e.g. Synchrotron radiation from supernova remnant

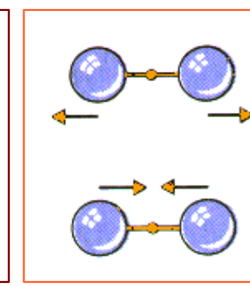


Atomic and molecular lines

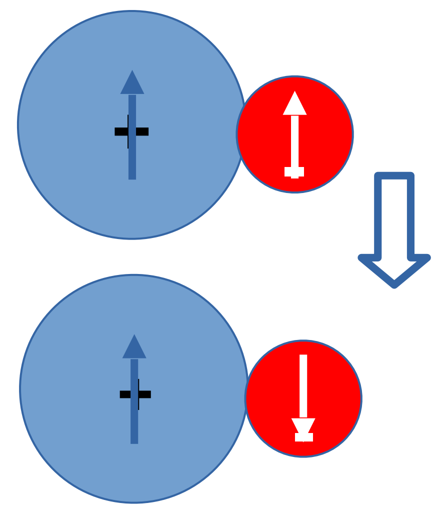


- Transitions at precise energy levels
  - Determined by quantum mechanics
    - Emission frequency $\nu = (E_{\rm upper} - E_{\rm lower})/h$
    - Large differences between electron orbital energies
      - High-frequency, short (optical/uv) λ emission/absorption
      - Molecular rotational transitions
        - Smaller energy differences
        - Longer (radio) λ emission/absorption
        - Microwave transitions in
        - excited vibrational states



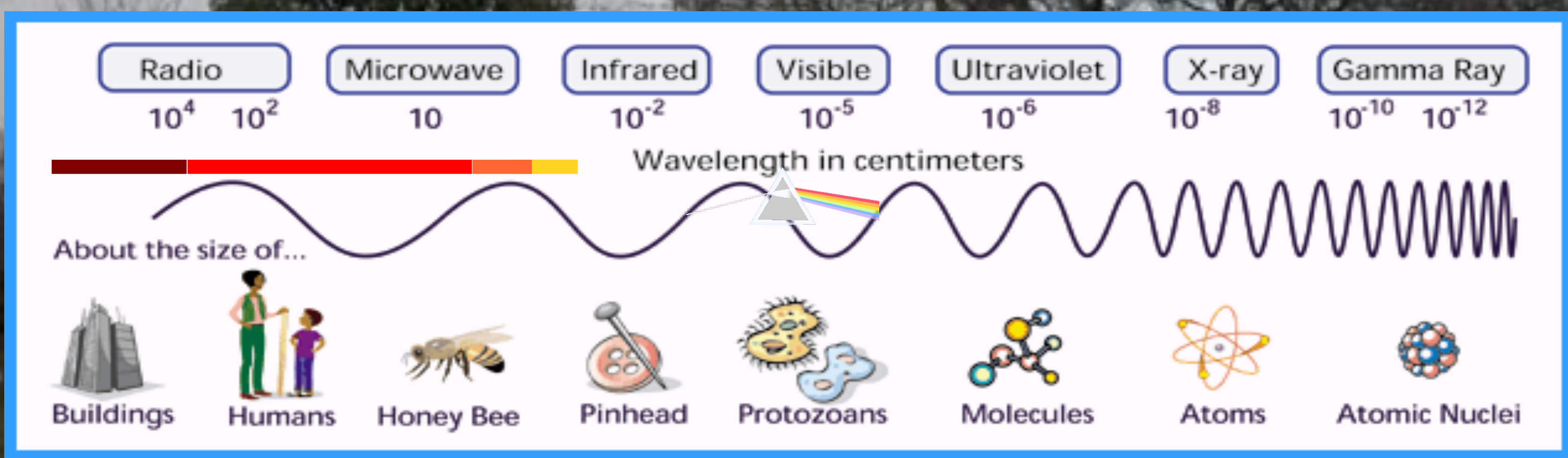


- Hydrogen atom proton and **electron**
  - Parallel or anti-parallel spins
  - Small energy difference gives λ 21-cm emission

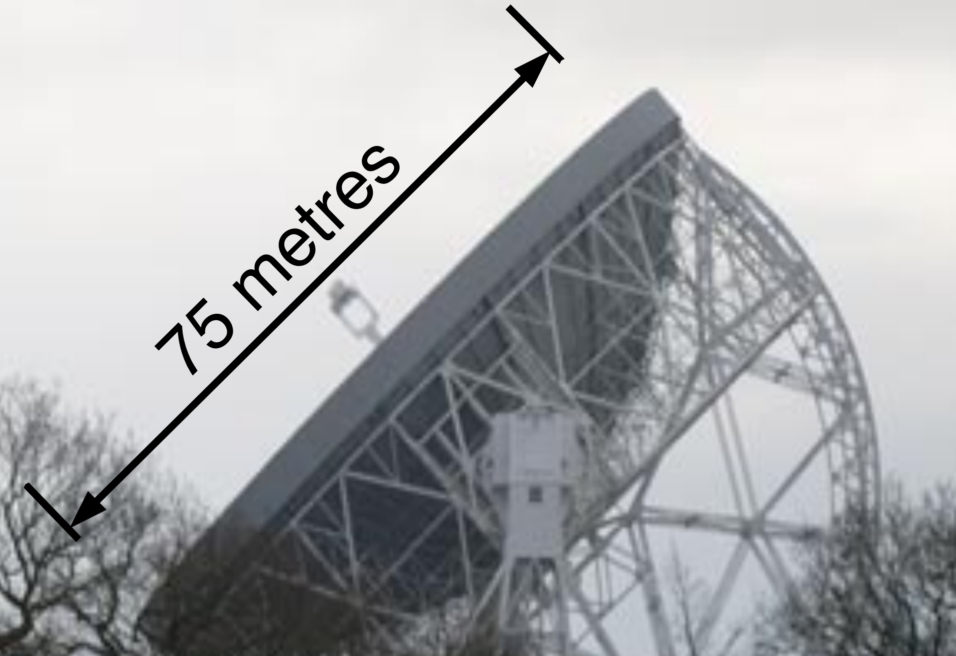


Long wavelengths pass through clouds

- >20 metres stopped by ionosphere
- 20 m to 1 cm, observe from ground day and night!
- 1 cm to 1 mm, need dry sites
- 1 to 0.3 mm, need high, very dry sites
- IR/visible: 10 to 0.4 micro-metres
- Everything else: observe from space!



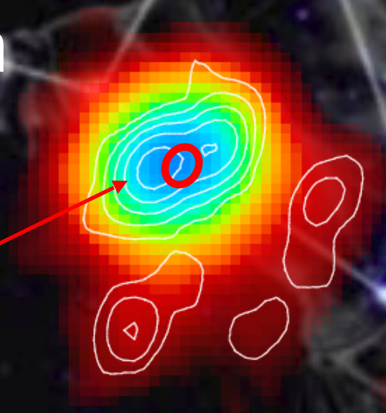
Resolution of a single telescope



- Lovell radio telescope, diameter *D* ~ 75 metres
- Observe radio waves at wavelength ~5 cm
- Resolution wavelength/*D* ~ 2.3 minutes of arc
    - 1/13 of the Moon's diameter

Radio telescopes & interferometry

- Many linked radio telescopes e.g. *e*-MERLIN array
- Maximum distance apart *B* = 217 km

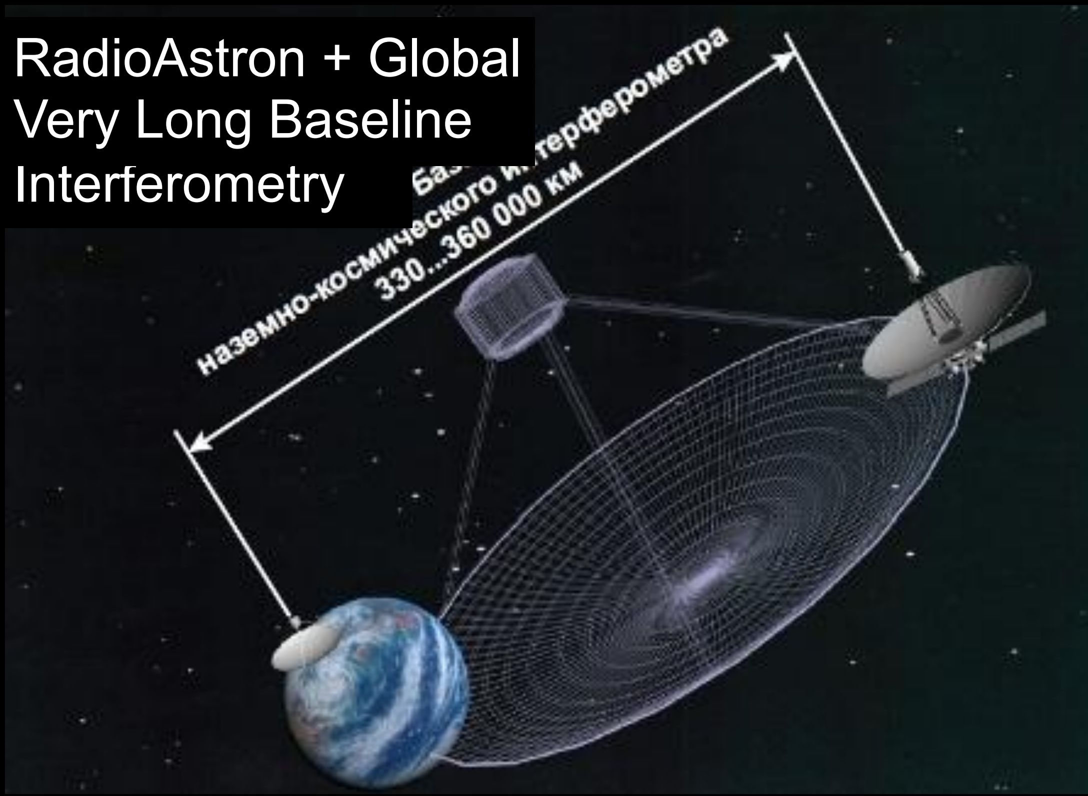


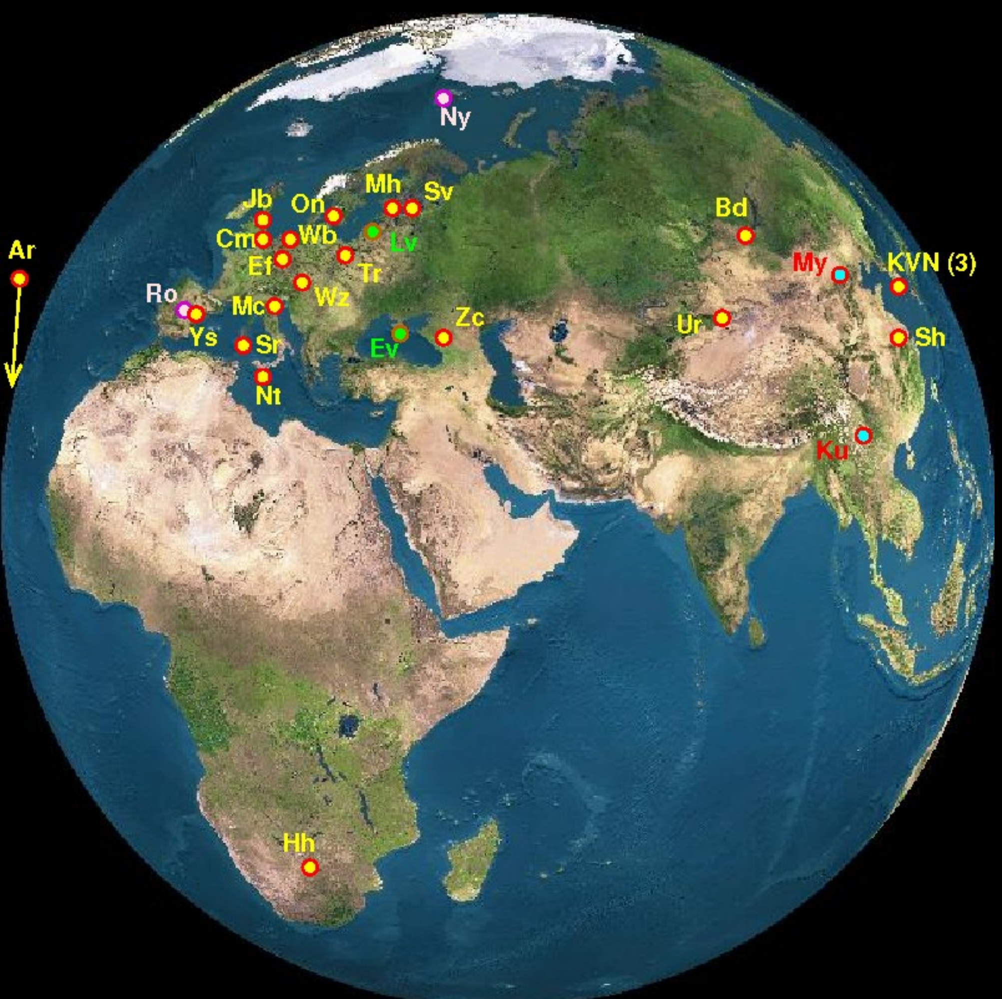
- Resolution (finest detail) wavelength/*B* ~ 0.045 arc seconds
  - A football pitch on the moon!
  - Starspots on Betelgeuse

Radio telescopes & interferometry

- To see even smaller details...

RadioAstron + Global Very Long Baseline Interferometry

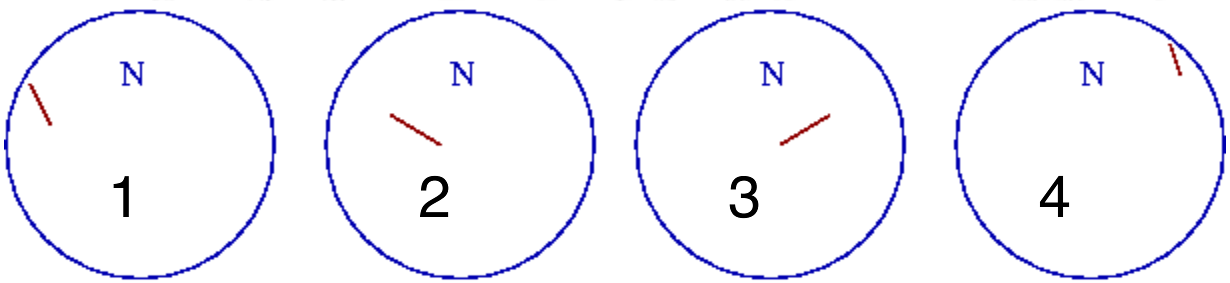


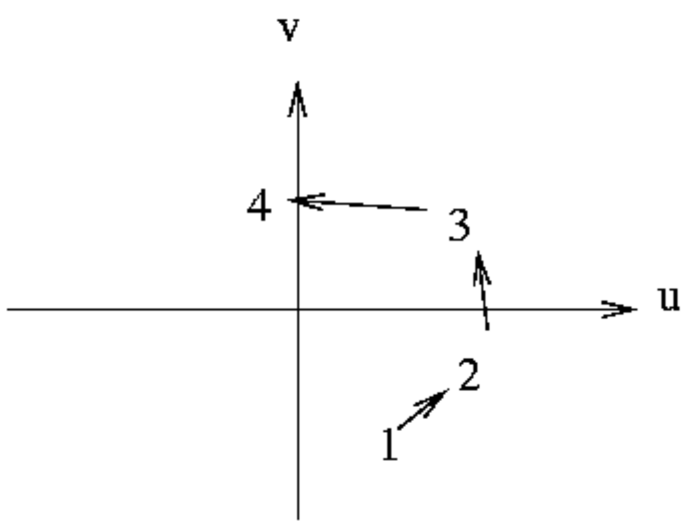


- Link radio telescopes all over the world
  - And in space

Earth Rotation Aperture Synthesis

1 



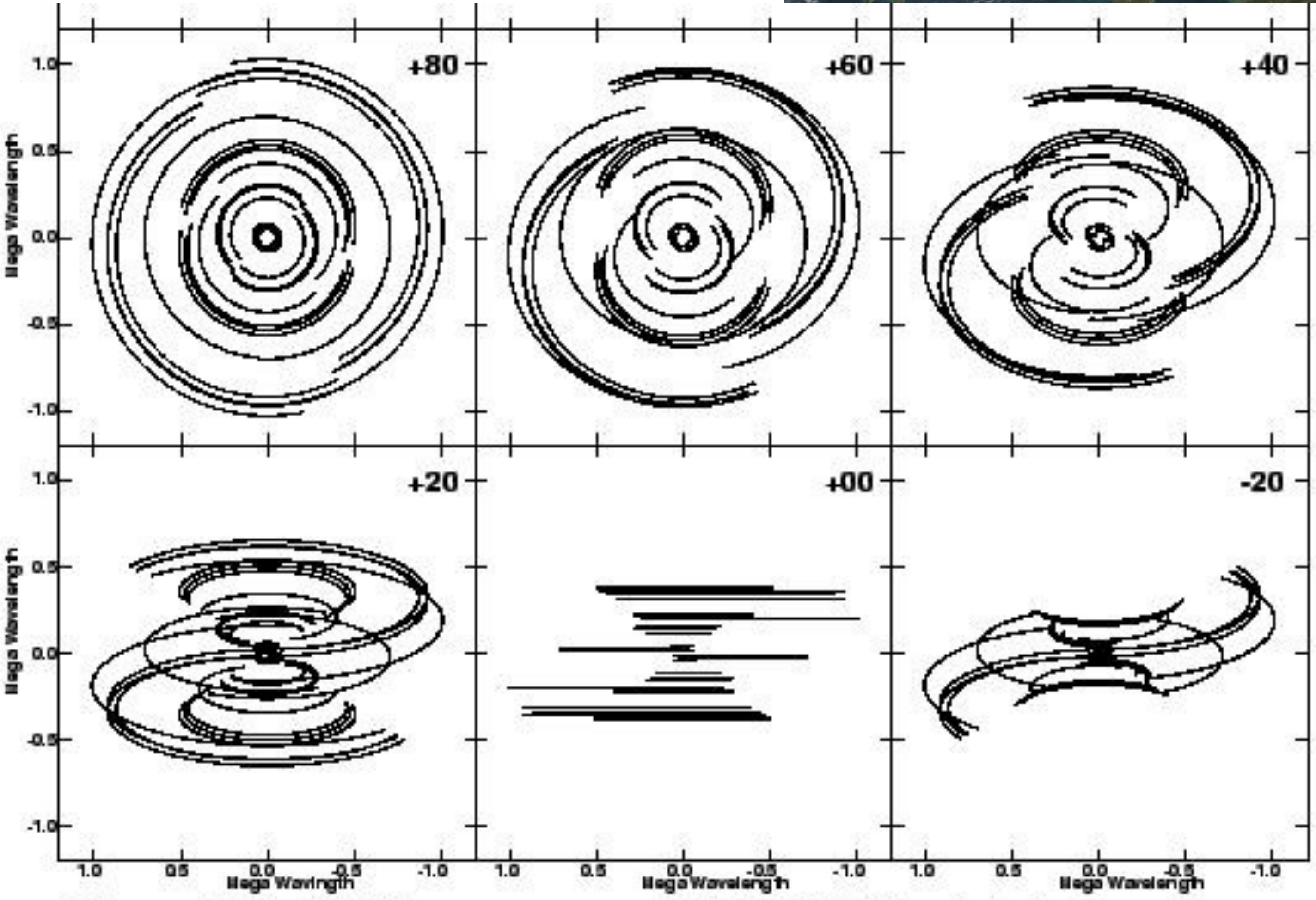


- Single baseline between antenna pair, in brown
- Vector sweeps round 180^{o} position angles/day
  - Projected length changes too
- Locus is an ellipse in *uv* plane
  - Samples range of sky scales/angles

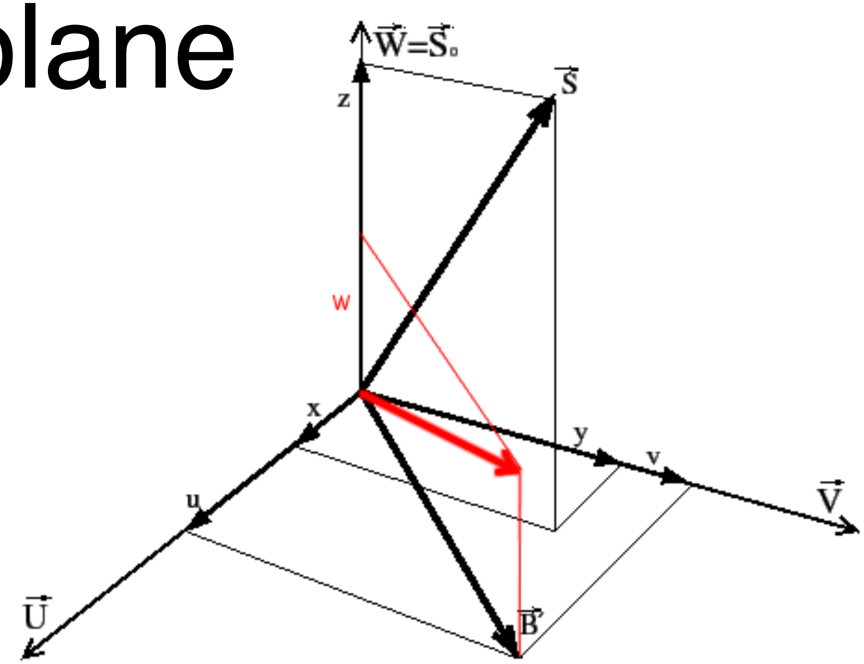
uv plane ellipses

- e-MERLIN telescopes latitude ~53^{o}
- Low Declination sources have

- Shorter tracks
- Poor North-South coverage



The *uv* plane

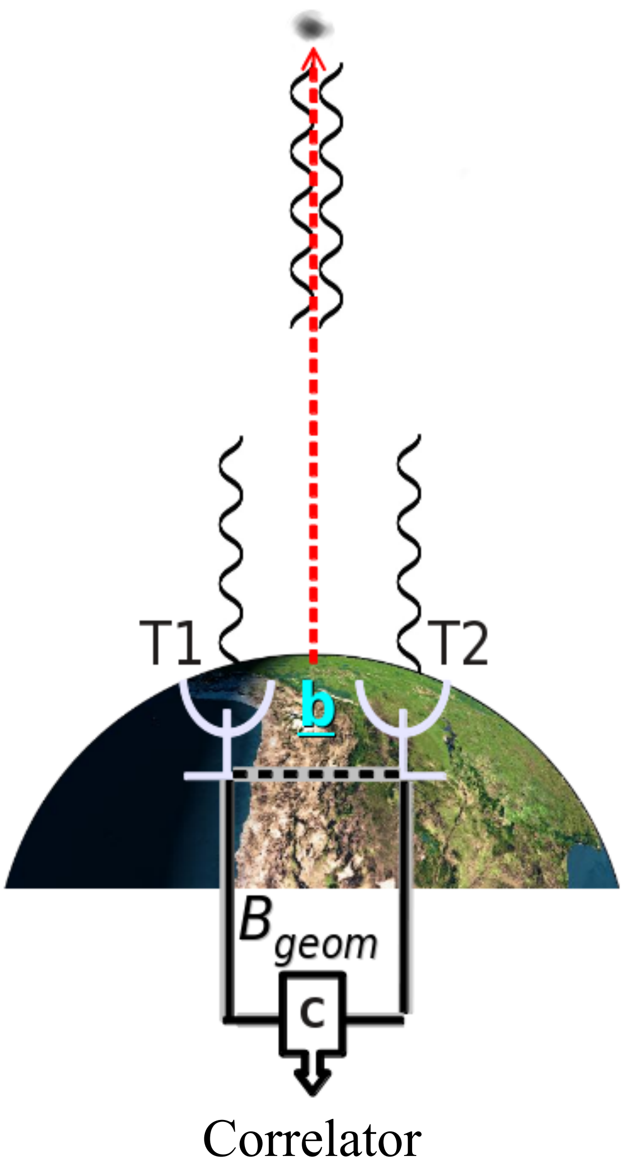


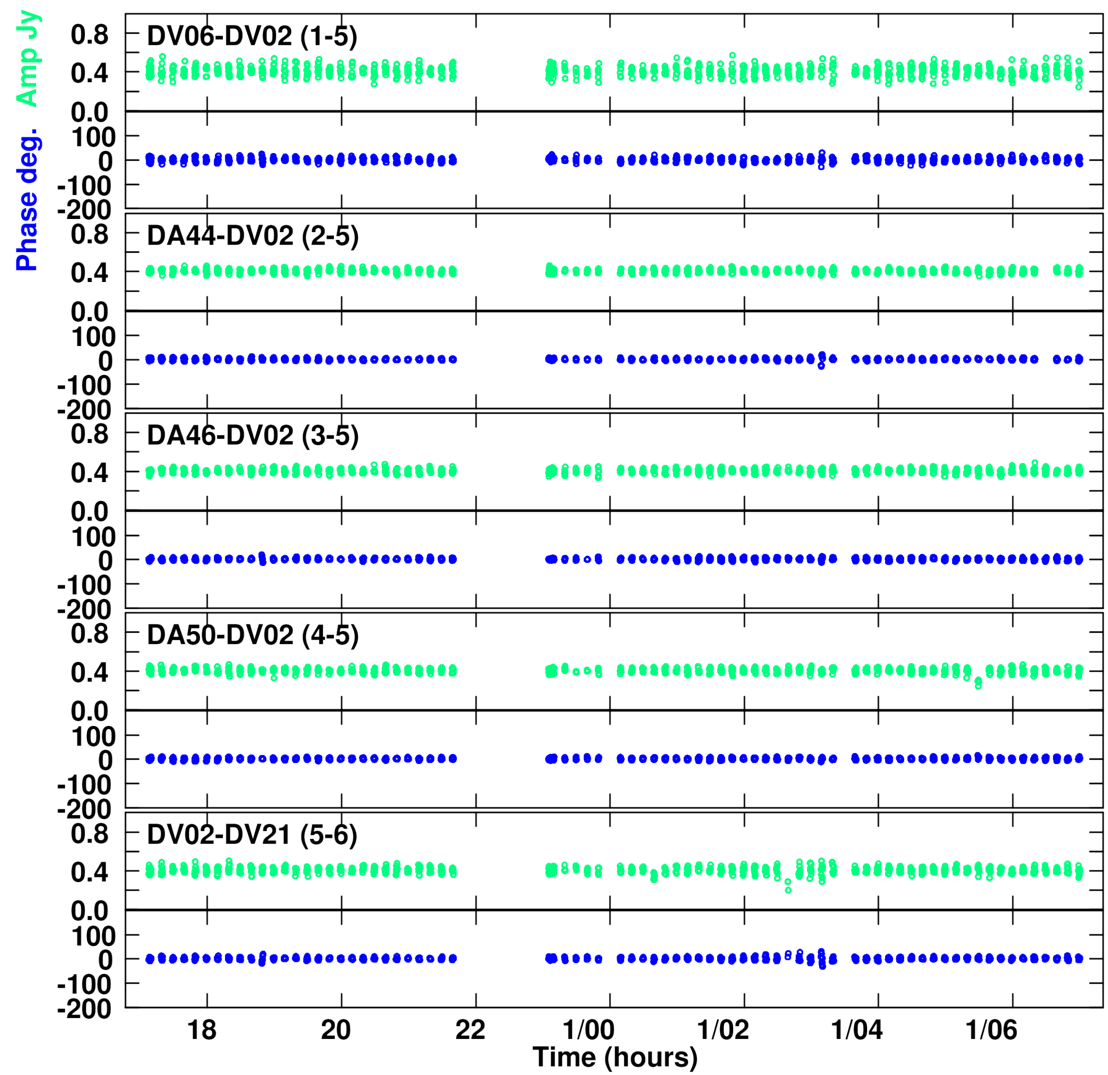
- x, y are celestial offsets corresponding to RA, Dec.
- **B** is the baseline vector
  - Orthogonal directions z, w ignored for small B, FoV
- Celestial reference position coords H_0, d_0

$$\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{w} \end{pmatrix} = \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0 \cos H_0 & \sin\delta_0 \sin H_0 & \cos\delta_0 \\ \cos\delta_0 \cos H_0 & -\cos\delta_0 \sin H_0 & \sin\delta_0 \end{pmatrix} \begin{pmatrix} B_X \\ B_Y \\ B_Z \end{pmatrix}$$

- Earth rotates: change of apparent source position (h,d)
  - u = B cos(d) sin(H_0 -h)
  - v = B sin(d)cos(d_0) – cos(d)cos(d_0)cos(H_0 -h)

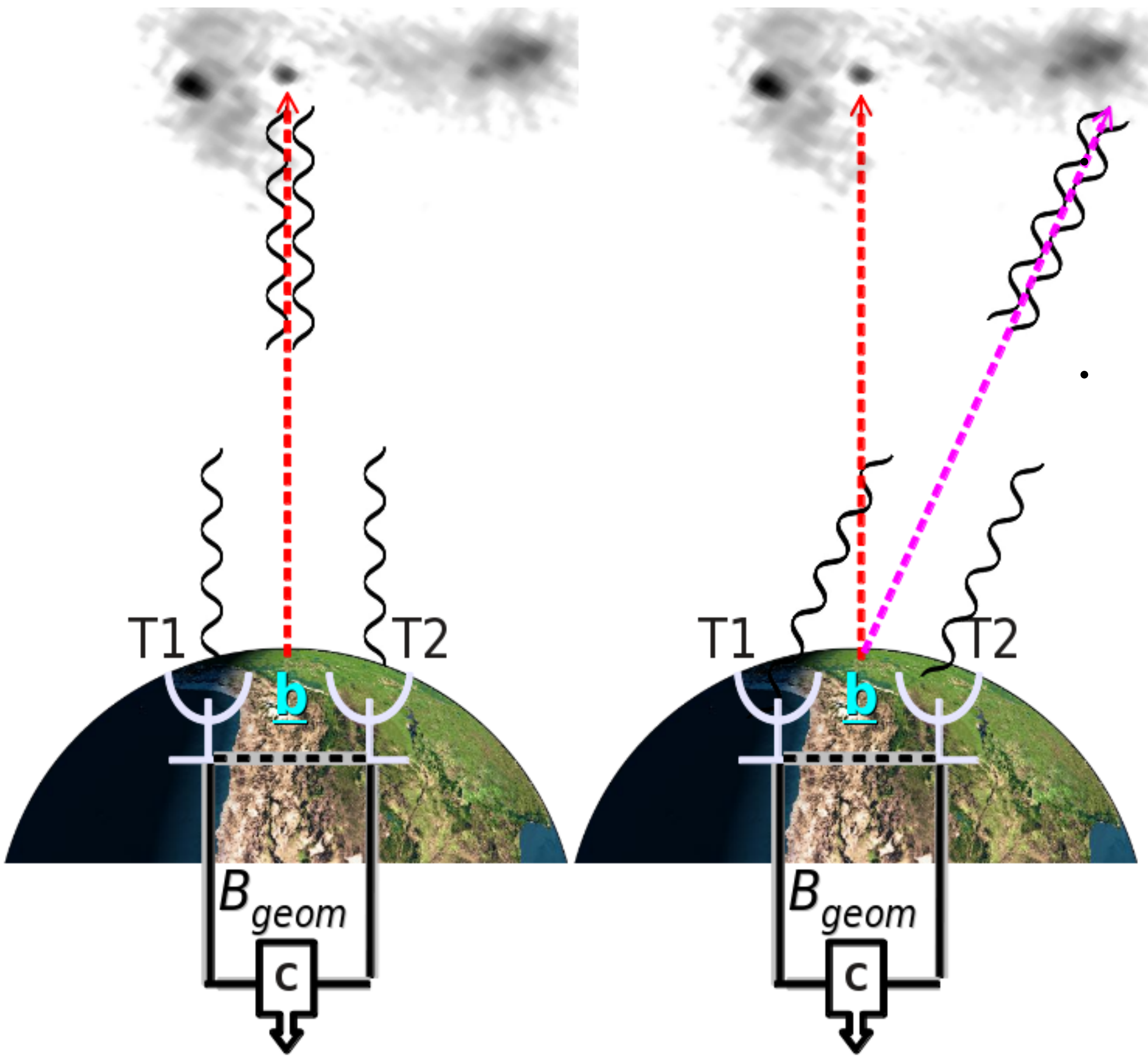
Point source overhead





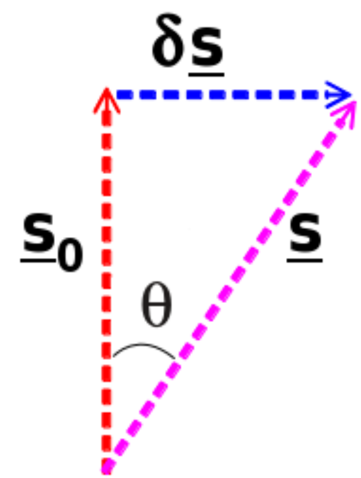
- Signals in phase, interfere constructively
  - Combined phase 0º, amp constant

Resolved source overhead



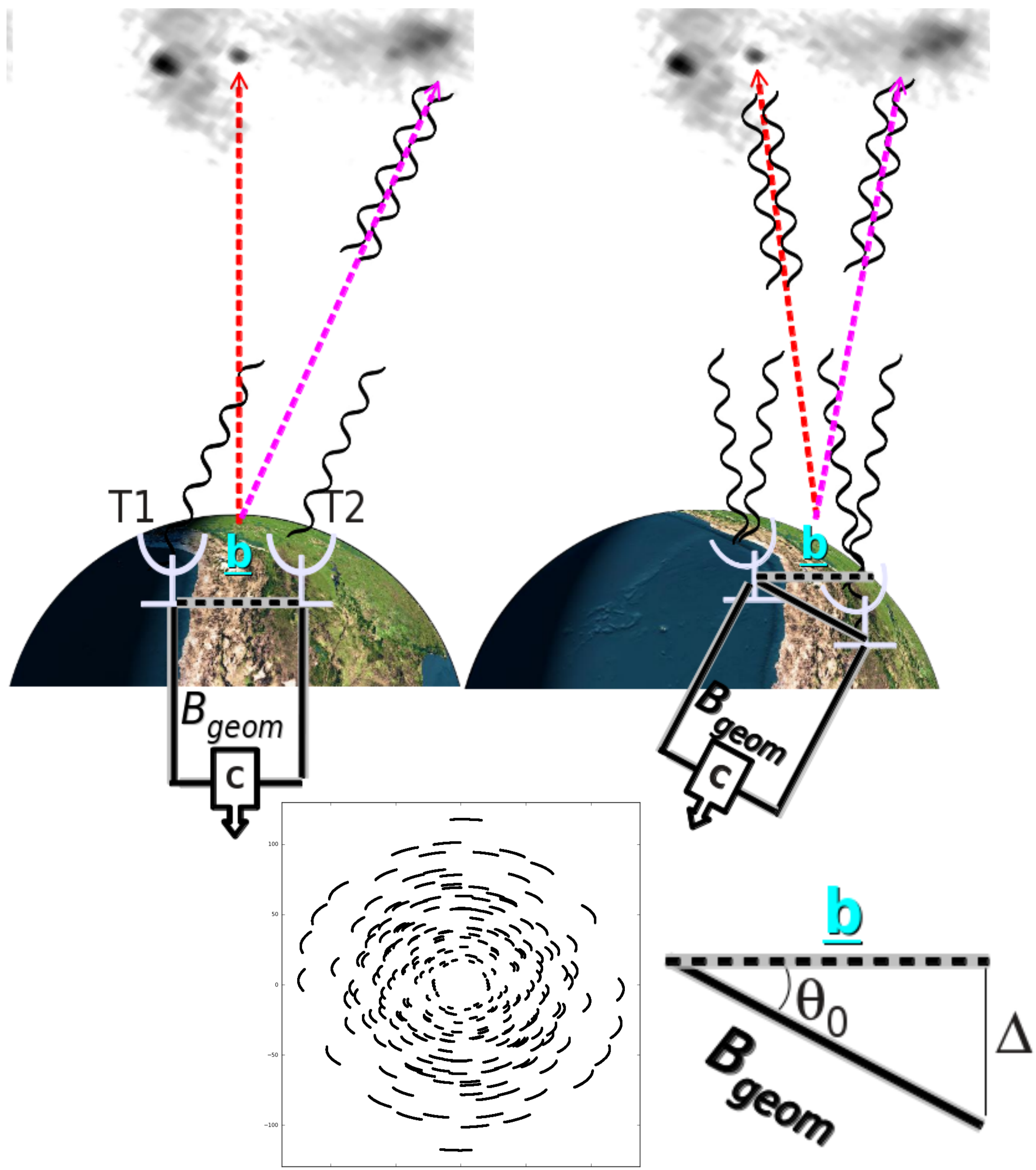
Signals from overhead vector $\mathbf{s_0}$

- Signals from lobe
  - Angular offset vector $d\mathbf{s}$
  - Path length $\mathbf{s} = d\mathbf{s} \cdot \mathbf{s_0}$



- Combined ϕ depends on $d\mathbf{s}$
- Complex visibility amplitude is sinusoidal function of ϕ

Earth rotates

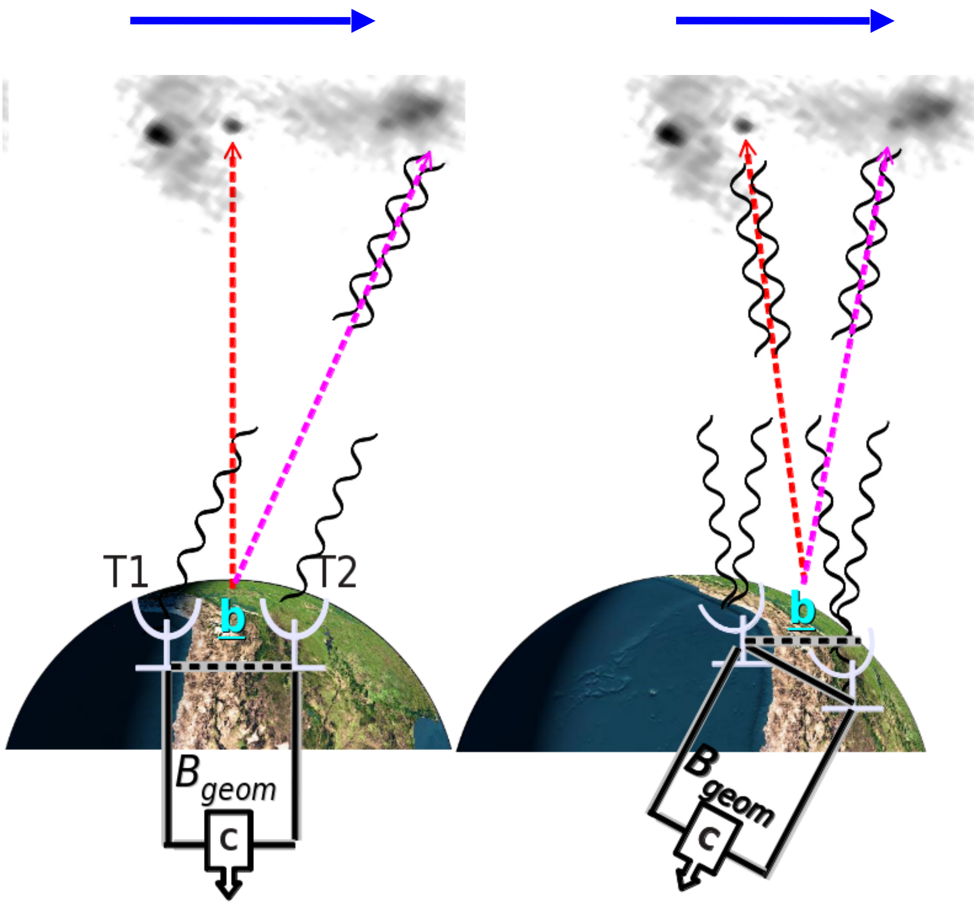


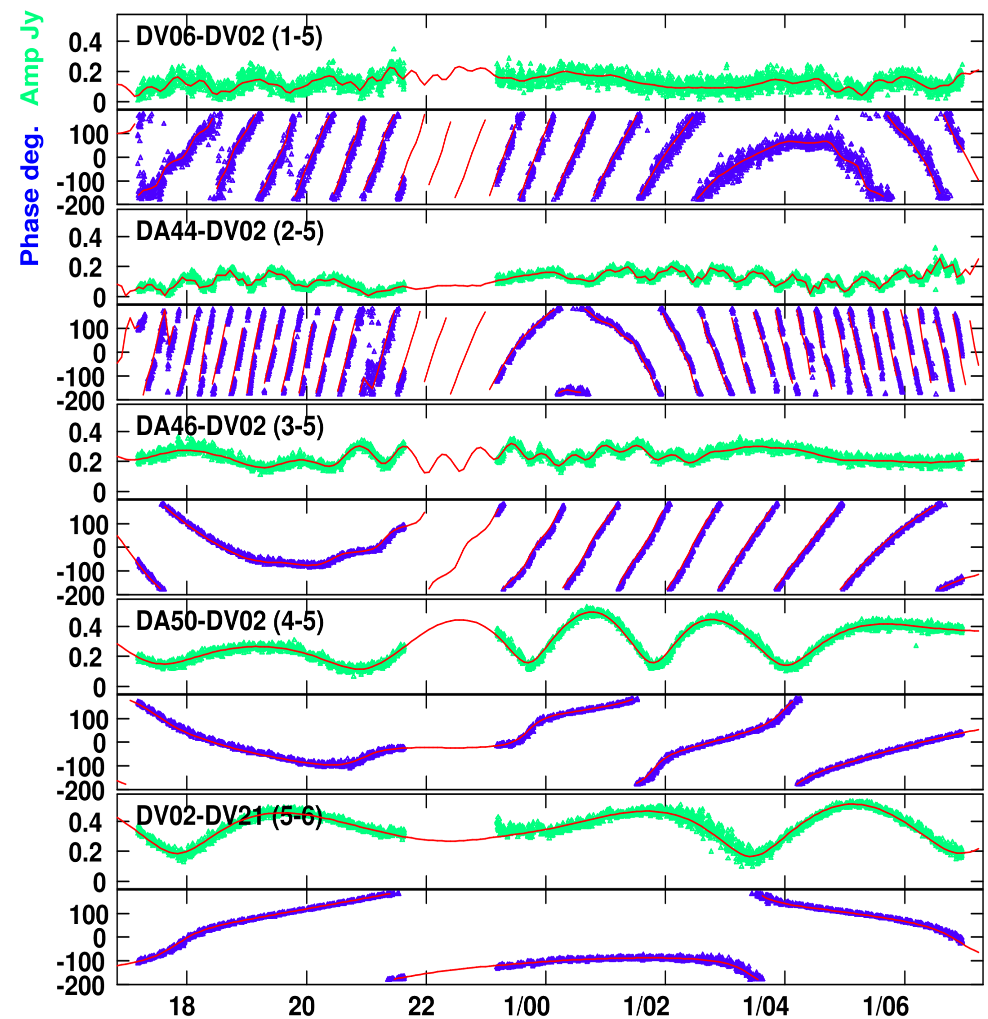
- Telescopes separated by baseline B_{geom}
- Earth rotates
  - Projected separation:

$$b = B_{\text{geom}} \cos(\theta_0)$$

- Samples different scales of source
- Additional geometric delay path Δ
  - Remove in correlator

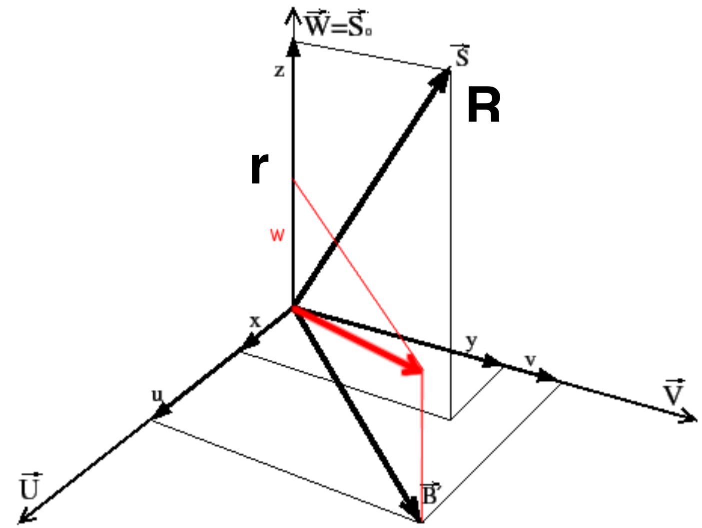
Earth rotation aperture synthesis





- Combined f depends on **ds** (*time*)
- Complex visibility amplitude is sinusoidal function of ϕ

An idealised interferometer



- Astrophysical source located at **R**
  - Observer at **r**
- Electromagnetic waves from **R**

$$E(\mathbf{R},t) = \int E_\nu(\mathbf{R}) e^{2\pi i\nu t} d\nu$$

  - Recall that this describes electric field as a sinusoidal function of time $e^{2\pi i\nu t} = \cos(2\pi\nu t) + i\sin(2\pi\nu t)$
  - $E_\nu(\mathbf{R})$ are complex, vector coefficients providing information about the structure of **R**
- Simplify (*will have to deal with complications later*):

  1. Implicitly assume emission is constant
  2. Ignore polarization – pretend $E_\nu(\mathbf{R})$ are scalar

An idealised interferometer

- Simplify (continued):

3. Radiation is monochromatic

$$\int E_\nu(\mathbf{r}) = \iiint P_\nu(\mathbf{R},\mathbf{r})E_\nu(\mathbf{R})dxdydz$$

. $P_\nu(\mathbf{R},\mathbf{r})$ is known as the Propagator

4. Sources are all very far away at fixed distance |**R**|

5. Radiation travel time from a point source at **R** to antennas at r_1, r_2 is the same, i.e. a wavefront enters both simultaneously

6. Radiation from R is not spatially coherent

Random noise: $\langle E_\nu(\mathbf{R_1})E^*_\nu(\mathbf{R_2})\rangle$ unless $\mathbf{R}_1$=$\mathbf{R}_2$

$\langle\rangle$ time average; E^*_ν complex conjugate of E_ν

An idealised interferometer

- Final simplification:

  7. Space between **r** and **R** is empty (no refraction etc.)

     - Using Huygens' principle, the propagator from area A at **R** becomes:

$$E_\nu(\mathbf{r}) = \int E_\nu(\mathbf{R}) \frac{e^{2\pi i|\mathbf{R}-\mathbf{r}|/c}}{|\mathbf{R}-\mathbf{r}|} dA$$

- Combining the signals arriving at $\mathbf{r}_1$, $\mathbf{r}_2$ gives the **correlation** of the radiation field at two observing locations,

$$C_\nu(\mathbf{r_1}, \mathbf{r_2}) = \langle E_\nu(\mathbf{r_1}) E_\nu^*(\mathbf{r_2}) \rangle$$

- Observed intensity: $I_\nu(\mathbf{s}) = |\mathbf{R}|^2 \langle |E_\nu(\mathbf{s})|^2 \rangle$

where **s** is the unit vector R/|**R**|

Spatial coherence function

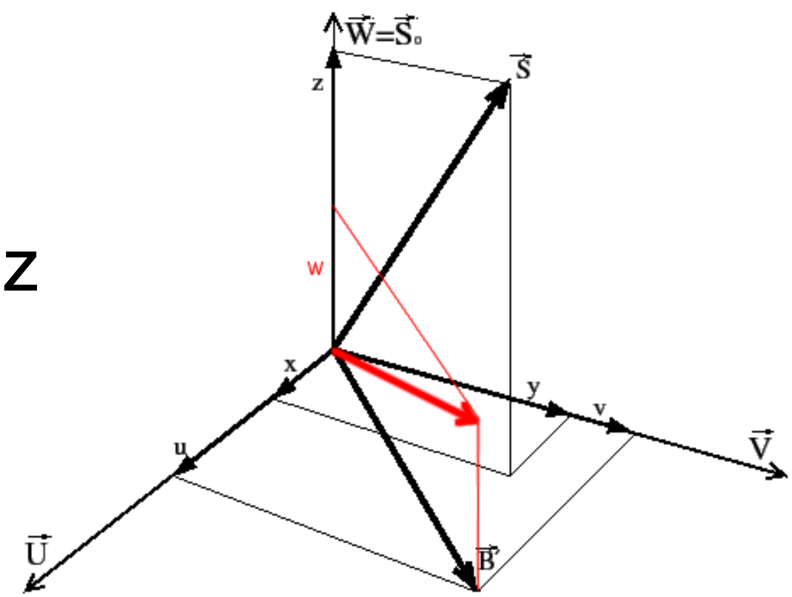
- Combining these expressions over a small region of solid angle Ω gives

$$C_\nu(\mathbf{r_1},\mathbf{r_2}) = \int I_\nu(\mathbf{s}) e^{-2\pi i \nu \mathbf{s}(\mathbf{r_1}-\mathbf{r_2})/c} d\Omega$$

- This is the spatial coherence function for radiation being measured by a detection array with two elements separated by $\mathbf{r}_1$-$\mathbf{r}_2$

regardless of where they are.

- Sky vector **s** has components x,y,z
  - where $z = \sqrt{1 - x^2 - y^2}$
    - (sometimes given as l,m,n)



Interferometer equation

- Baseline $\mathbf{r}_1$-$\mathbf{r}_2$ is expressed as a vector in the u,v,w plane, customarily in length units of wavelength l
- Substituting for **s** and $\mathbf{r}_1$-$\mathbf{r}_2$ in the spatial coherence function gives $V_\nu(u,v,w) = \iint I_\nu(x,y) e^{-2\pi i \frac{(ux+vy+wz)}{z}} dxdy$
  - For a small patch of sky, z~1
  - For a small, flat radio telescope array w=0
- This gives a simple definition of the Visibility Function V as a **Fourier Transform** relation with intensity I

$$V_\nu(u,v) = \iint I_\nu(x,y) e^{-2\pi i(ux+vy)} dxdy$$

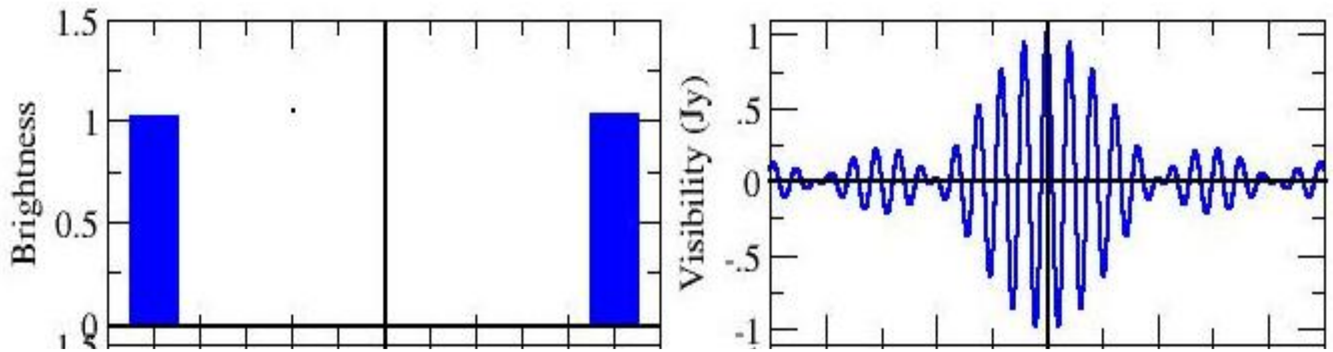
  - or, $V_\nu(u,v) \Leftrightarrow I_\nu(x,y)$ *(van Cittert-Zernicke theorem)*

Fourier Inversion

- The distribution of intensity on the sky is reconstructed by inverting the Fourier relation

$$I_\nu(x,y) = \iint V_\nu(u,v) e^{2\pi i(ux+vy)} dudv$$

- Additional terms such as antenna response can be included: $$V_\nu(u,v) = \iint A_\nu(x,y) I_\nu(x,y) e^{-2\pi i(ux+vy)} dxdy$$



One-dimensional Fourier relationships

Removing simplifications

Mitigate by observing and data reduction techniques:

1. Only very compact sources are time variable; divide the data into small segments (more sophisticated techniques under development)

2. Scalar approximation may be OK for unpolarised sources, or polarisation can be calibrated

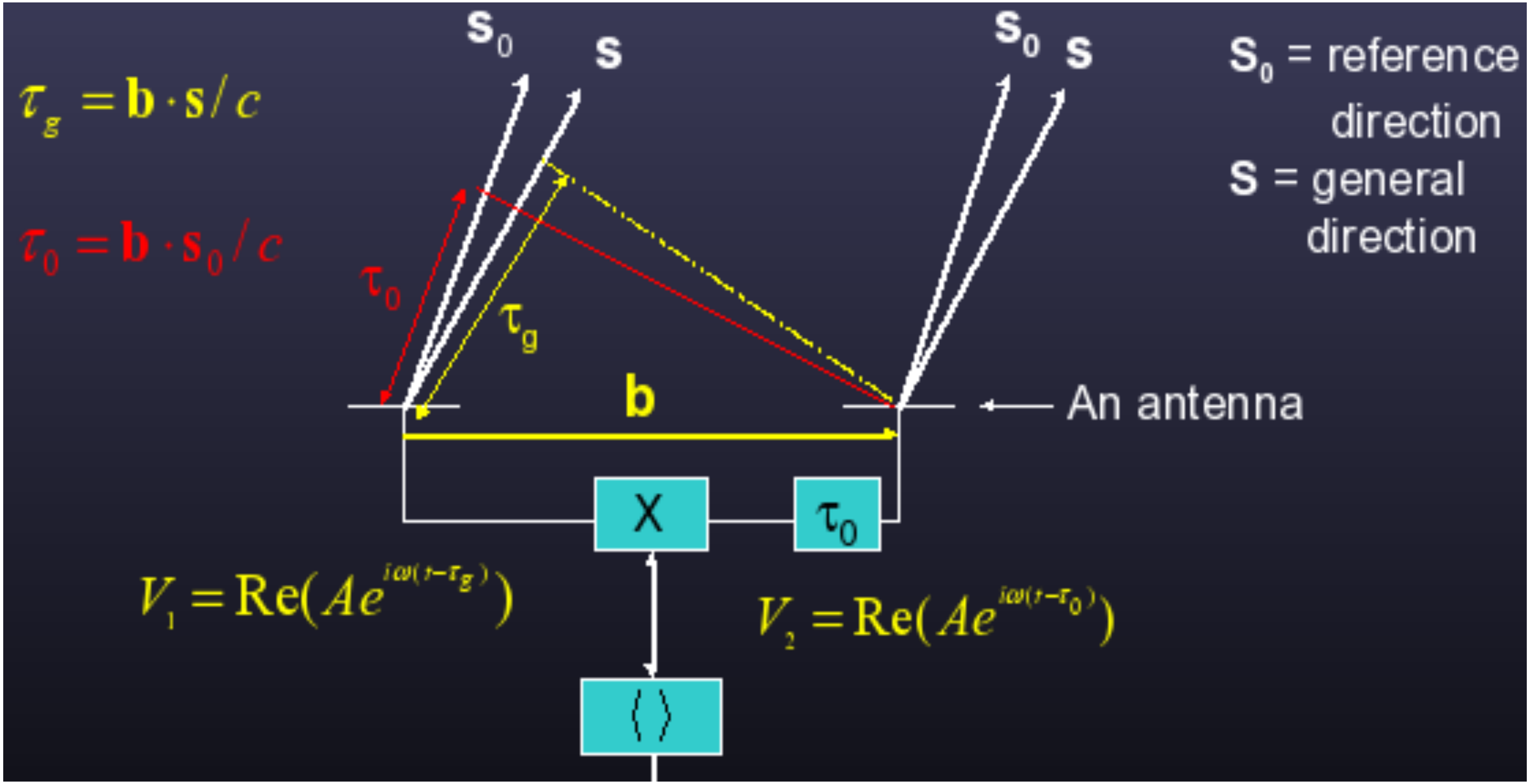
3. Multiple spectral channels, each treated as single ν

4, 5, 6. Allow for non-co-planar baselines in imaging wide fields. Special techniques for e.g. nearby planets

7. Space (and the atmosphere) do distort the signal – numerous calibration techniques!

- See the rest of this school!

Geometric (etc.) delay

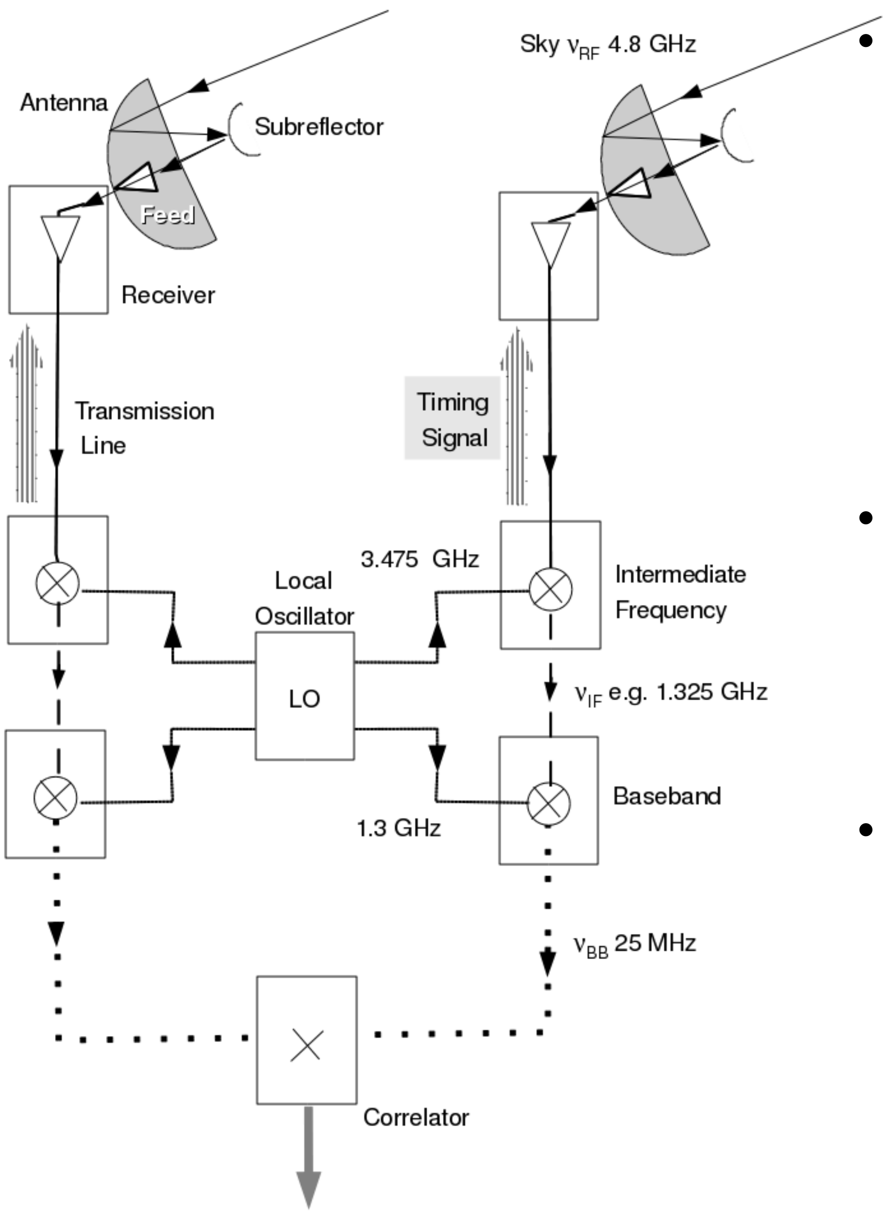


- Delay t corresponds to change in arrival time of wavefront
- Equivalent to a frequency-dependent phase change 2π τ ν

Delay correction

- Geometric delay t_g-t_0 and effect of Earth rotation
  - Different path lengths depending on direction on sky
    - Calculated and corrected as observations are made
      - Only precise for field centre; can limit field of view
- Differences in electronic paths
- Atmospheric effects
  - Bulk effects can be removed during observations; residuals tackled during calibration.
- Three different ‘field centres’, normally but not always set to be the same: pointing, delay and phase tracking.

Signal path



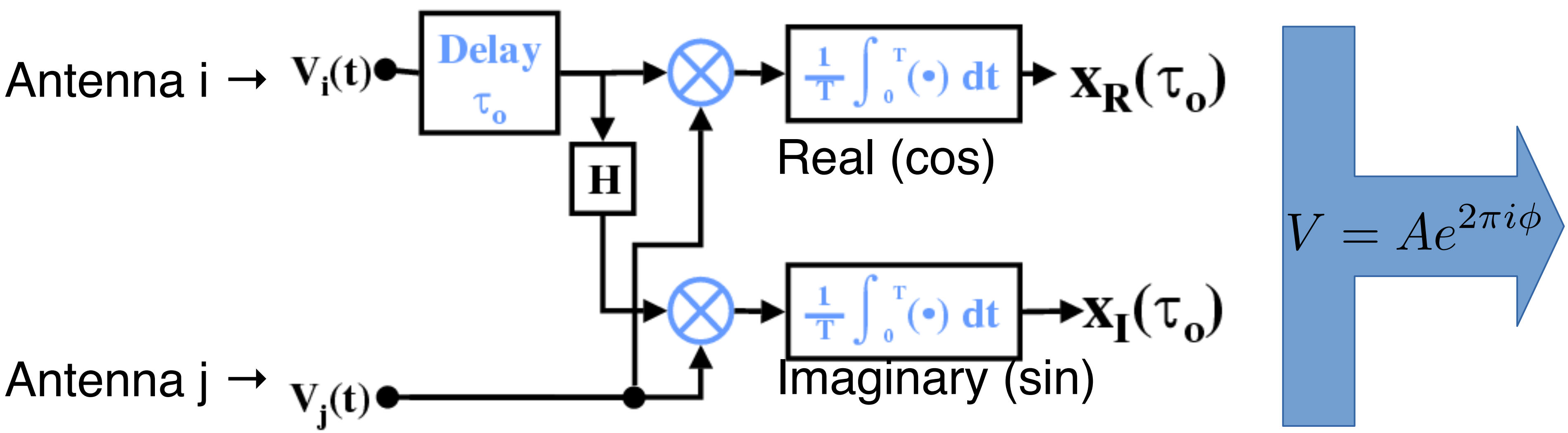
- Receiver/amplifier system
  - Fluctuating voltage
  - Mix with LO signal to frequencies where electronics are most sensitive
- Sampling
  - Nyquist rate: > $1/2\nu$
    - <0.5 ns at 1 GHz
- Quantisation
  - 2-bit for modest dynamic range
    - More bits for sensitivity/ resilience to interference

Correlation

- Correlator combines digitised signals from each pair of antennas (X) and Fourier Transforms them (F)
  - May be done in either order: XF or FX correlators
  - Output series of complex visibilities, amplitude A, phase ϕ

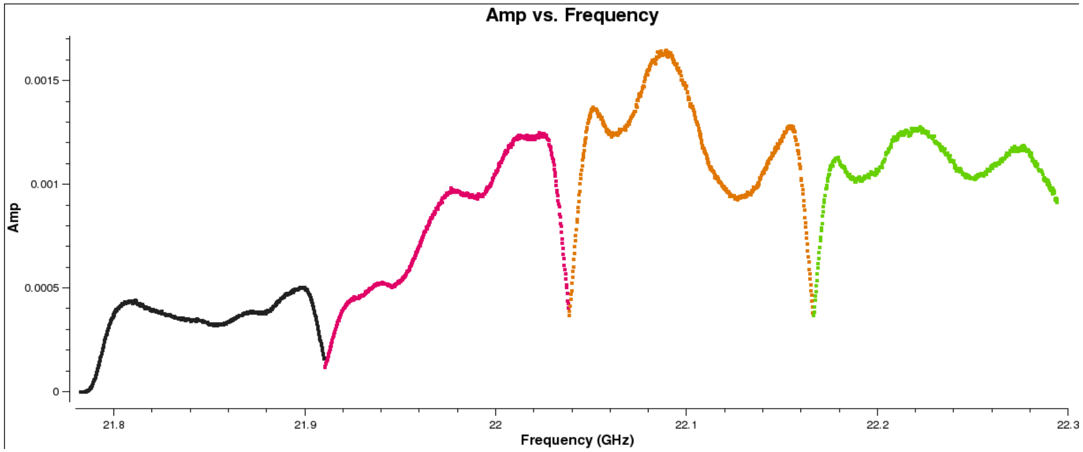
$$V_\nu(u,v) = \iint I_\nu(x,y) e^{-2\pi i(ux+vy)}\,dxdy \approx Ae^{2\pi i\phi}$$

  - Per integration, per baseline, per polarization, per spectral chan



Correlation

- Visibilities are averaged in time (typically 1 sec)
- Input polarizations correlated as parallel hands
  - e.g. LL, RR (to make total intensity)
    - Cross-hands LR, RL for polarization optional
- Input often digitally 'filtered' into sub-bands (spw)
  - Hardware and digital effects, including the FT of sharp edges to sinc functions, produces non-linear bandpass



Bandpass response for 4 spectral windows (spw) also known as IFs

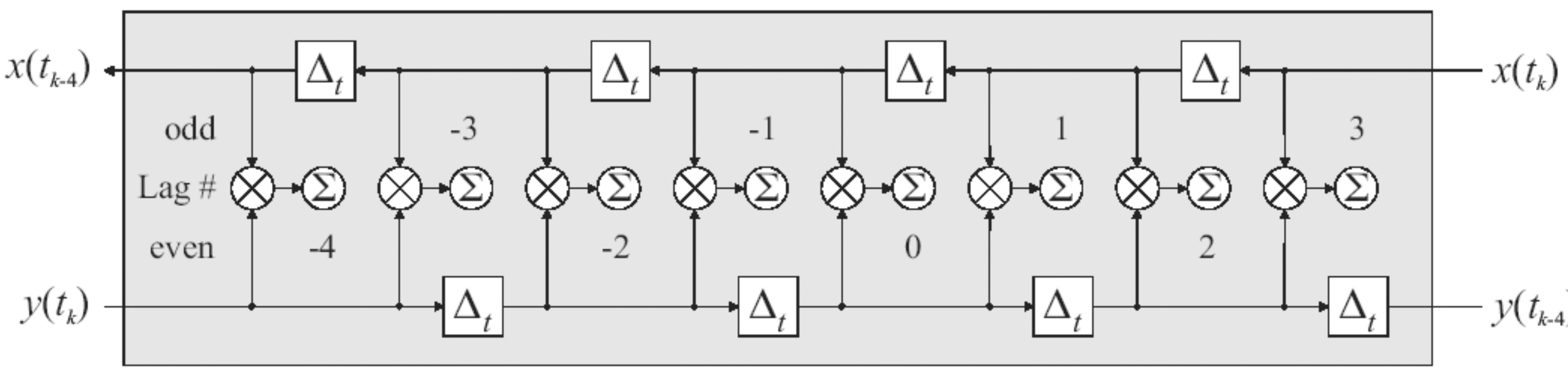
Correlation

- Each spectral window has many frequency channels
  - Formed digitally: in XF correlator, before combination:
    - Add series of lags [-N, -(N-1)..0..(N-1)]d*t* to each sample
      - FT transforms time to frequency domain

$$V(u,v,t) = \int V(u,v,\nu)e^{2\pi i t\nu}d\nu$$

(Weiner-Khinchin theorem)

    - Number of channels 2^N (power of 2)
      - Spacing $d\nu = 1/2dt$



Sensitivity

- Noise floor limited by system temperature

$$T_{\rm sys} = \frac{1}{\eta_A e^{-\tau_{\rm atm}}}[T_{\rm Rx} + \eta_A T_{\rm sky} + (1-\eta_A)T_{\rm amb}]$$

contributions from Receiver, sky and 'ambient' (hardware & ground temperature). Antenna area $A_{\rm eff}$, efficiency n_A

- Noise of data taken by array is given by

$$\sigma_{\rm sys} = \frac{T_{\rm sys}}{\eta_A A_{\rm eff}\sqrt{(N(N-1)/2)\ \Delta\nu\Delta t N_{\rm pol}}}$$

N antennas, frequency span $\Delta\nu$, time span Δt, $N_{\rm pol}$ Rx pols

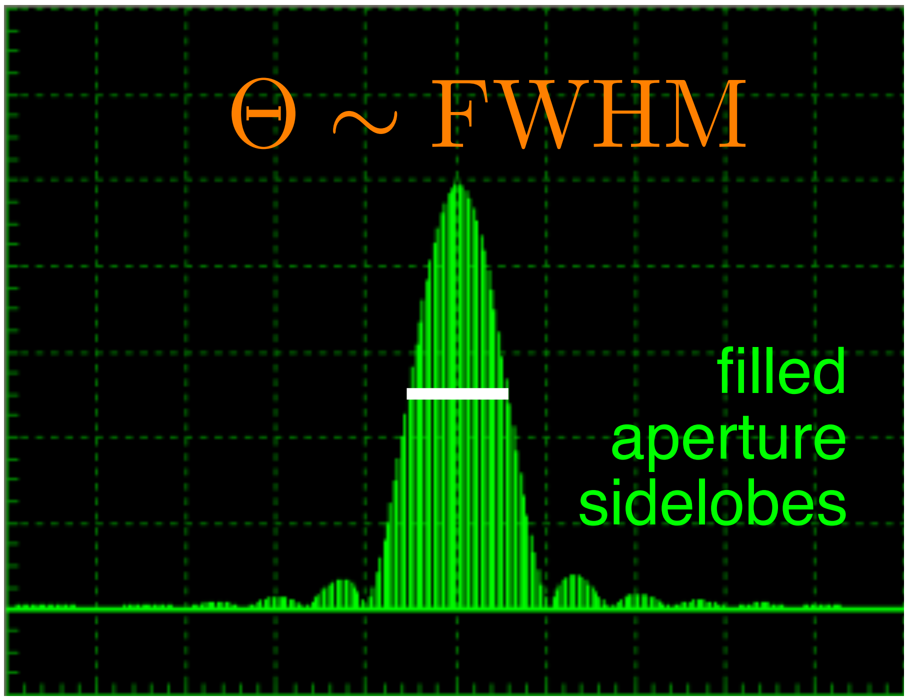
- Opacity, and hence noise, increases with zenith angle

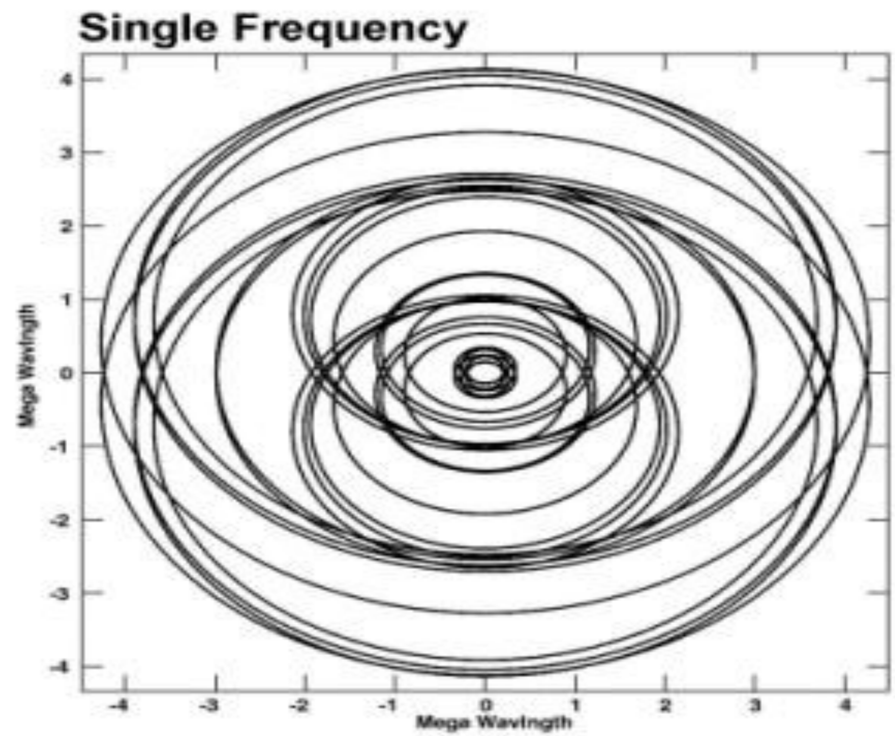
$$T_{\rm received} = T_{\rm source}e^{\tau_{\rm atm}/\cos(z)} + T_{\rm atm}(1-e^{\tau_{\rm atm}/\cos(z)})$$

atmospheric absorption emission

Interferometer sidelobes

- One antenna: maximum resolution $\Theta = \lambda / D$
  - *D* = 25 m at 21 cm (1.4 GHz) gives $\Theta \sim 0.5^{\circ}$

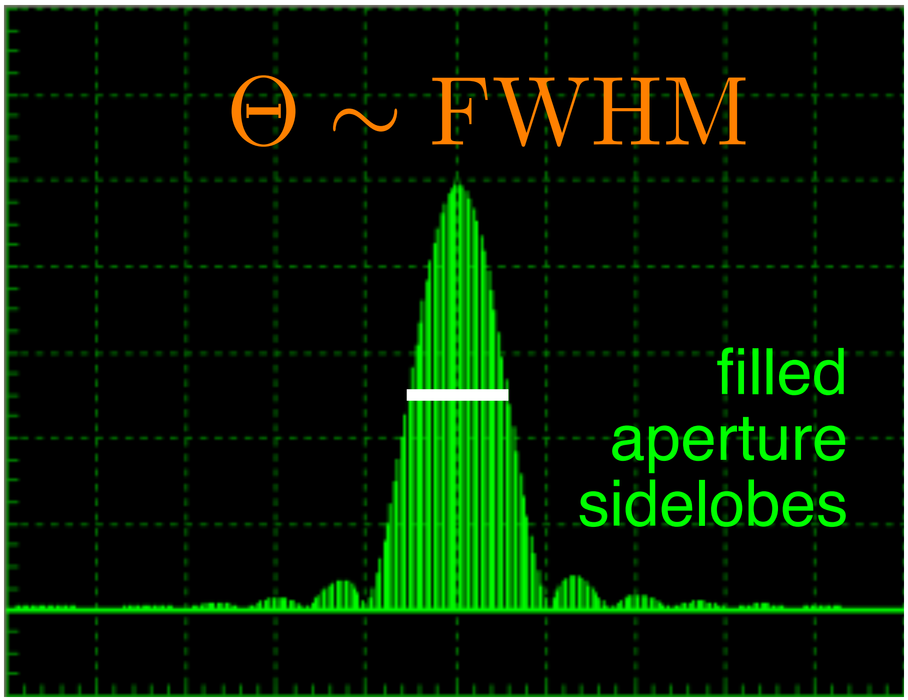


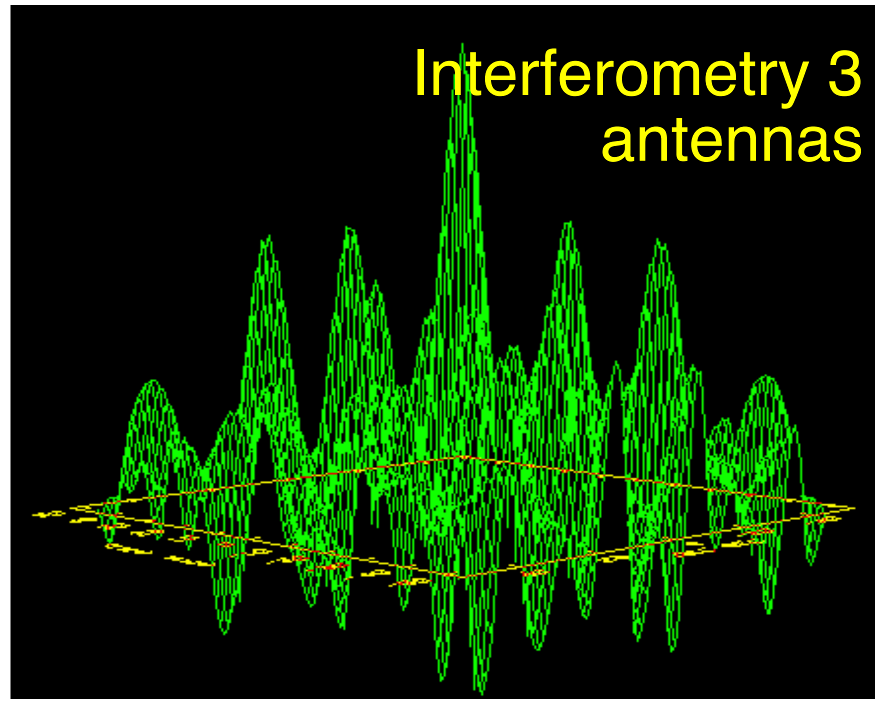


- Many antennas:
  - Maximum resolution $\theta = \lambda / B$
  - *B* ~200 km at 21 cm gives $\theta \sim 0.2$ arcsec

Interferometer sidelobes

- One antenna: maximum resolution $\Theta = \lambda / D$
  - D = 25 m at 21 cm (1.4 GHz) gives $\Theta \sim 0.5^{\circ}$

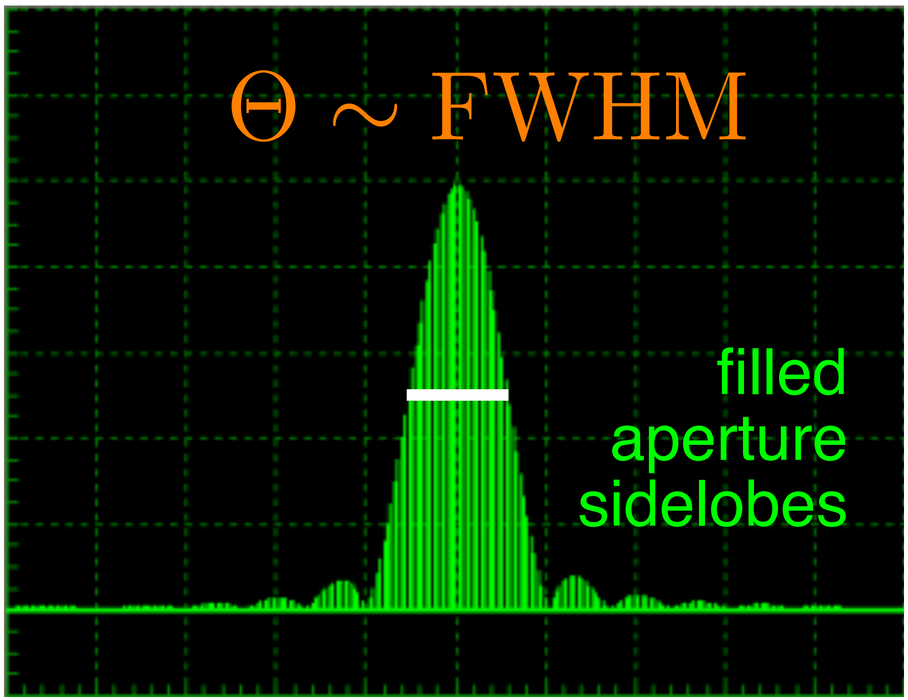


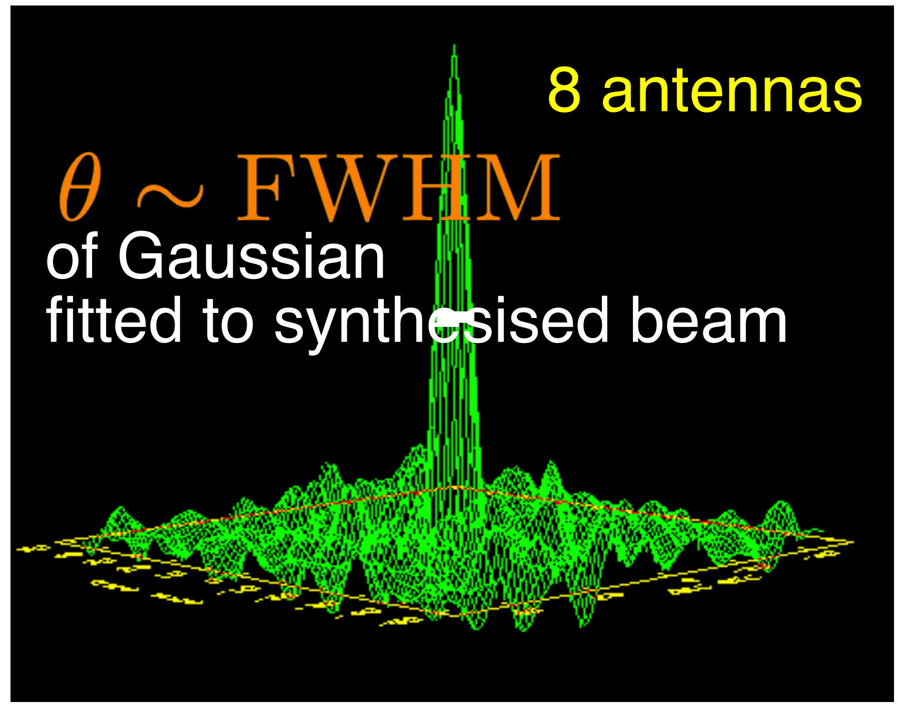


- Many antennas:
  - Synthesised beam is Fourier transform of uv tracks
  - Gaps in uv coverage make sidelobes in beam

Interferometer sidelobes

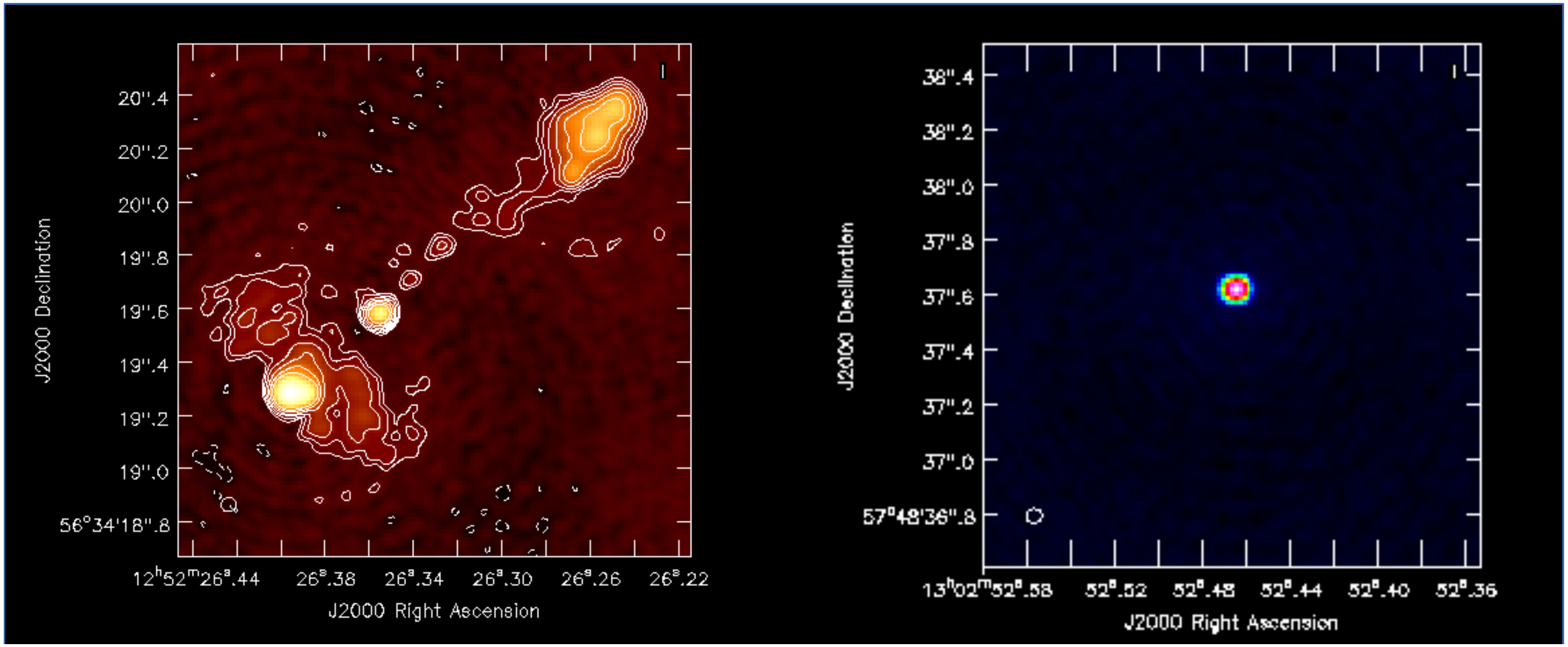
- One antenna: maximum resolution $\Theta = \lambda / D$
  - D = 25 m at 21 cm (1.4 GHz) gives $\Theta \sim 0.5^\circ$



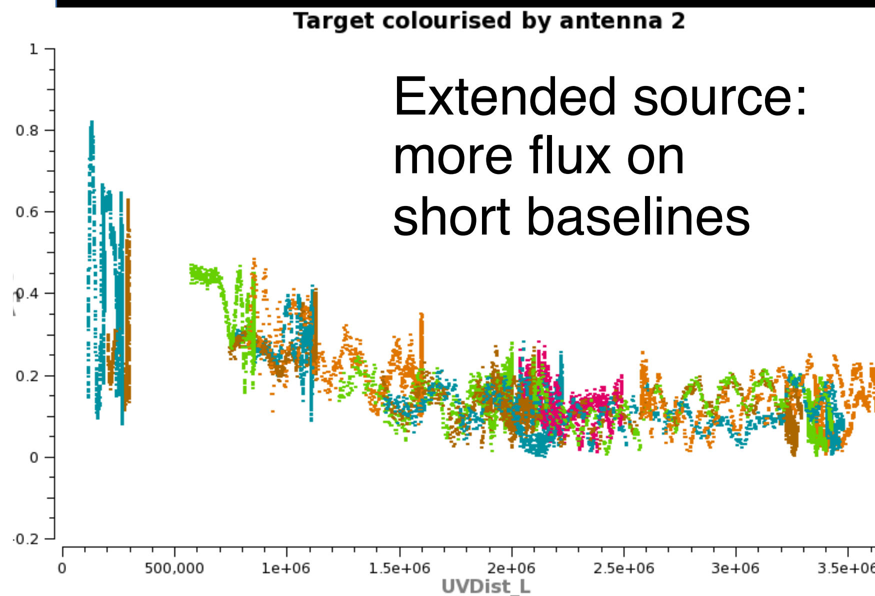


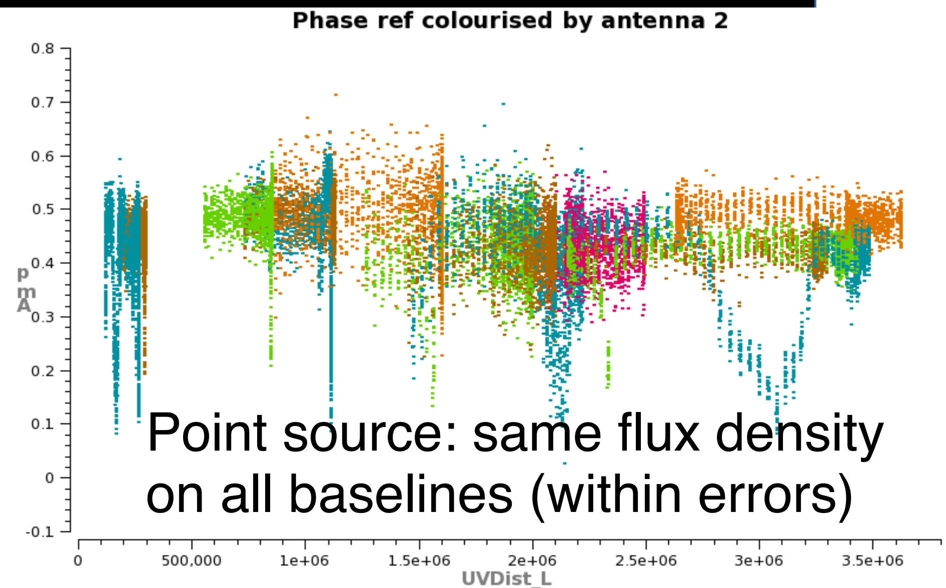
- Many antennas:
  - Maximum resolution $\theta = \lambda / B$
  - More antennas means fewer sidelobes

Source structure in *uv* plane



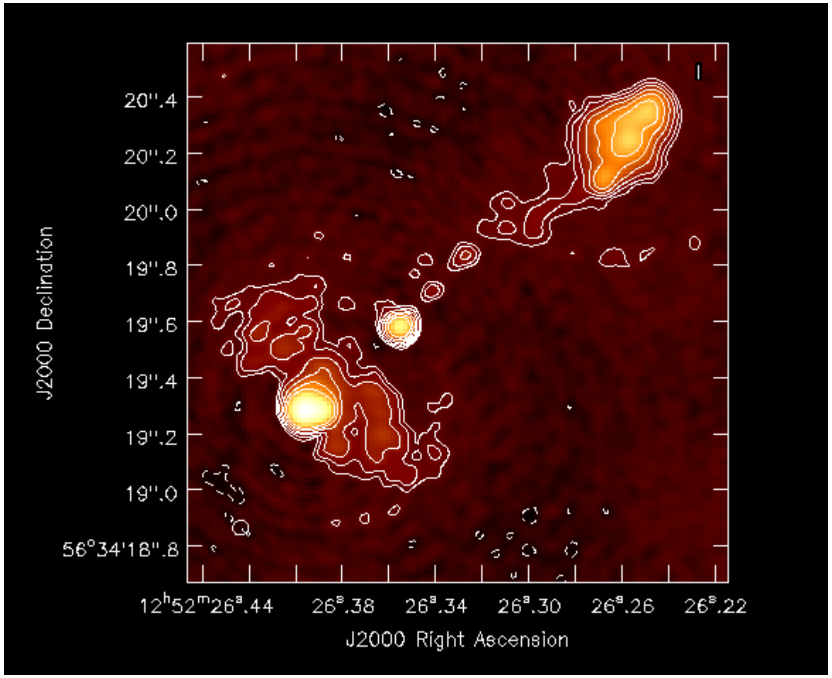
Visibility amplitudes

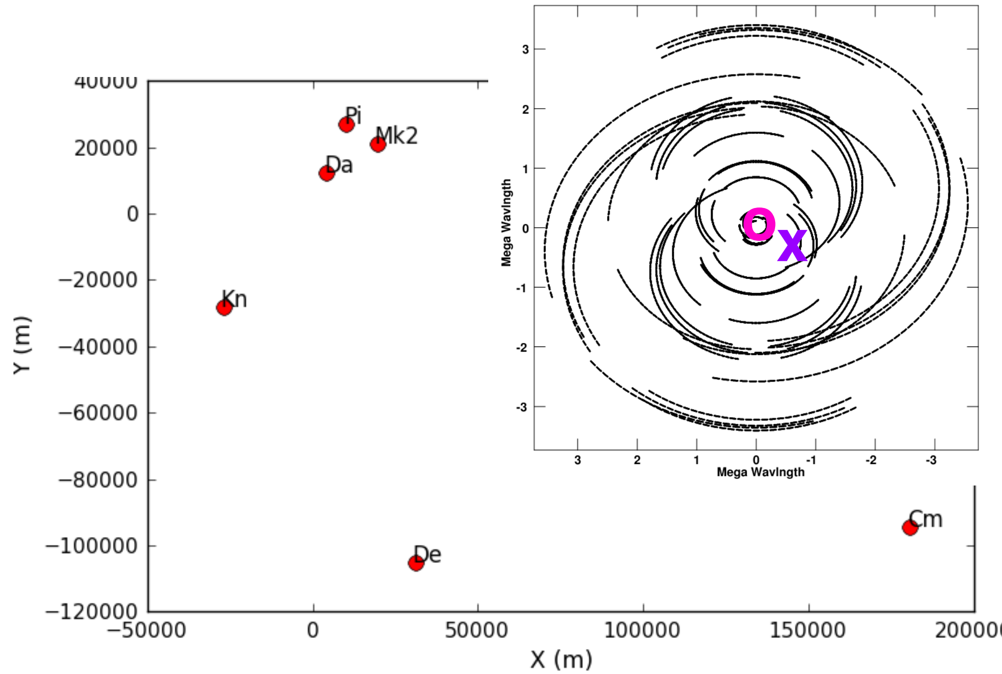


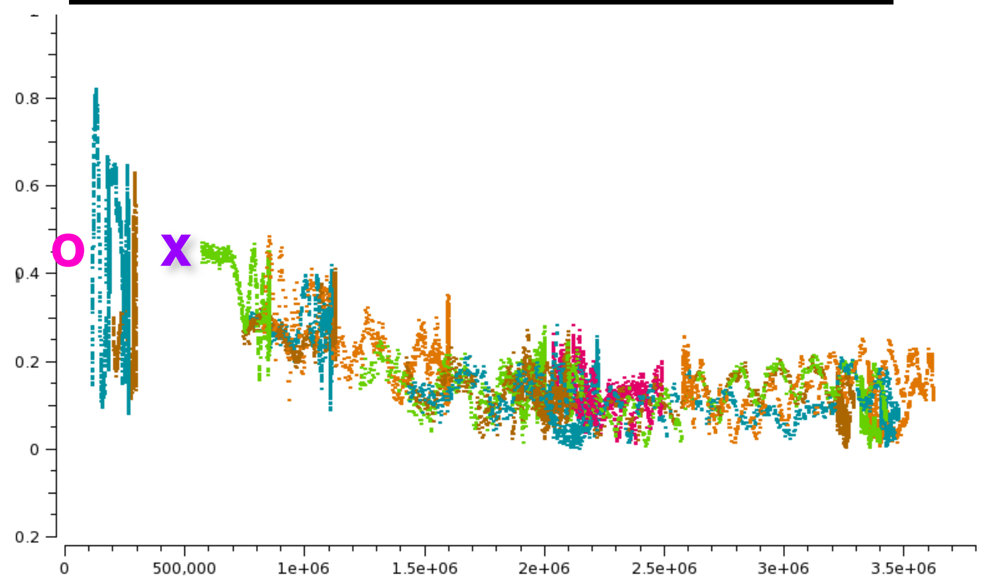


Baseline length in wavelengths (*uv* distance)

Source structure in *uv* plane







Visibility amplitudes

Baseline length in wavelengths (*uv* distance)

Limited range of antenna separations

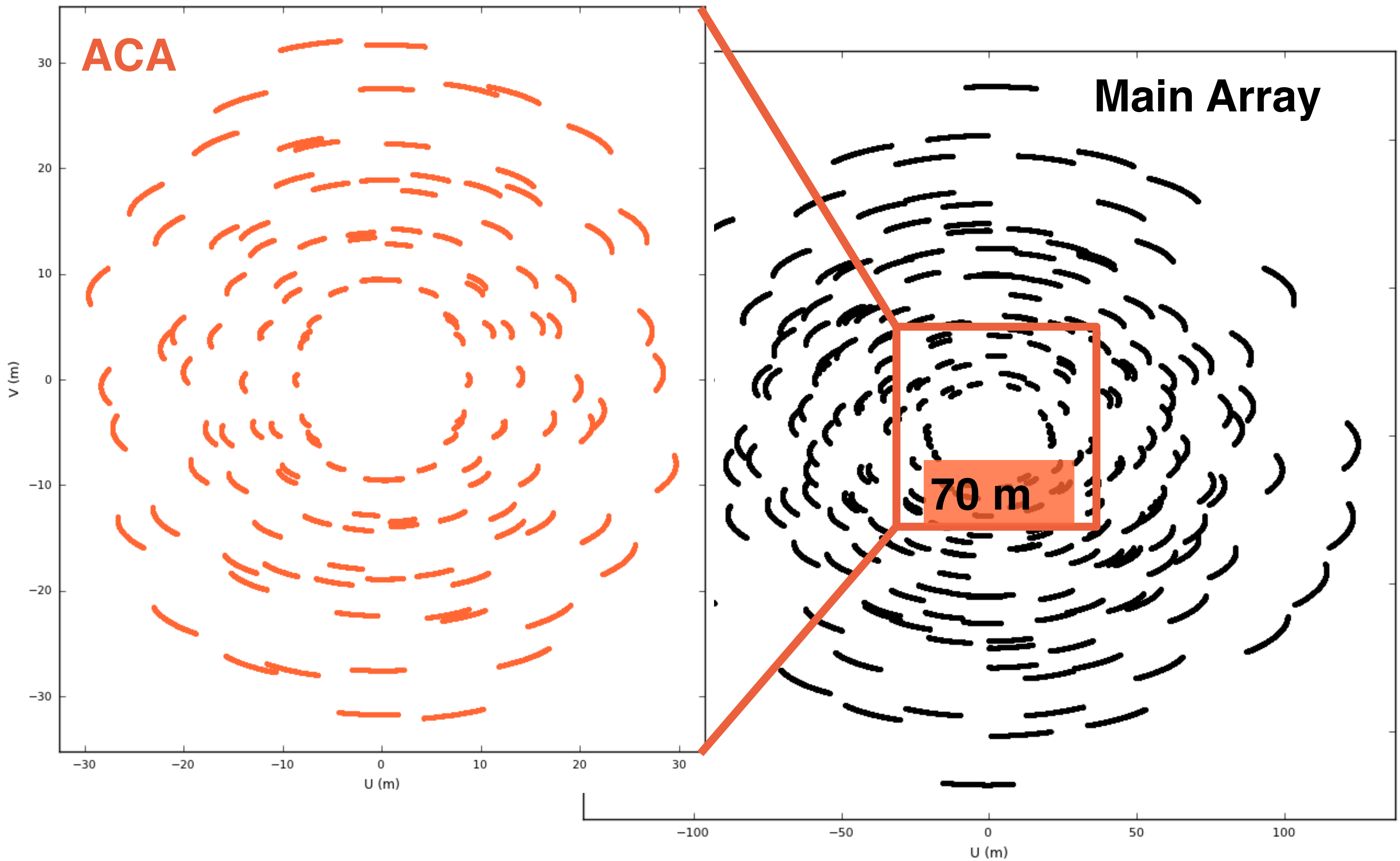
Gaps in uv coverage

Some spatial scales not sampled, central hole

Interpolate – but residual sidelobes

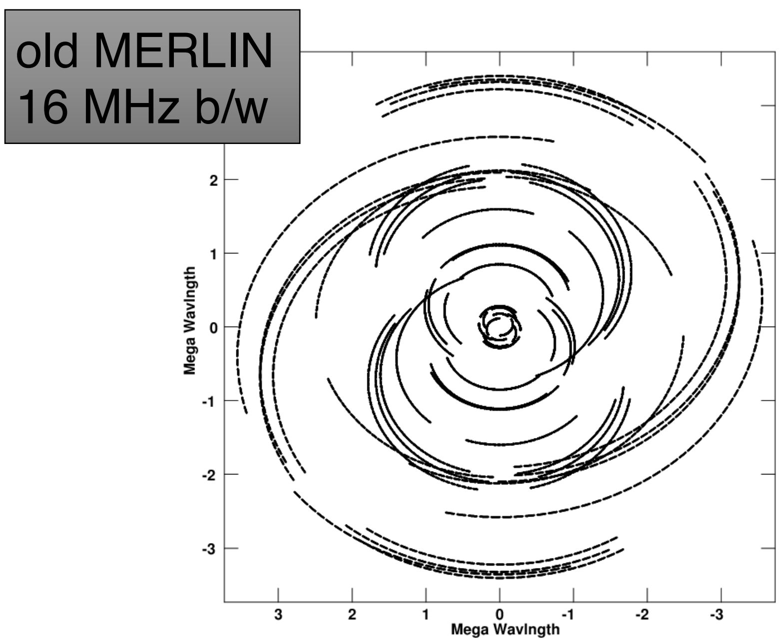
Large scales invisible

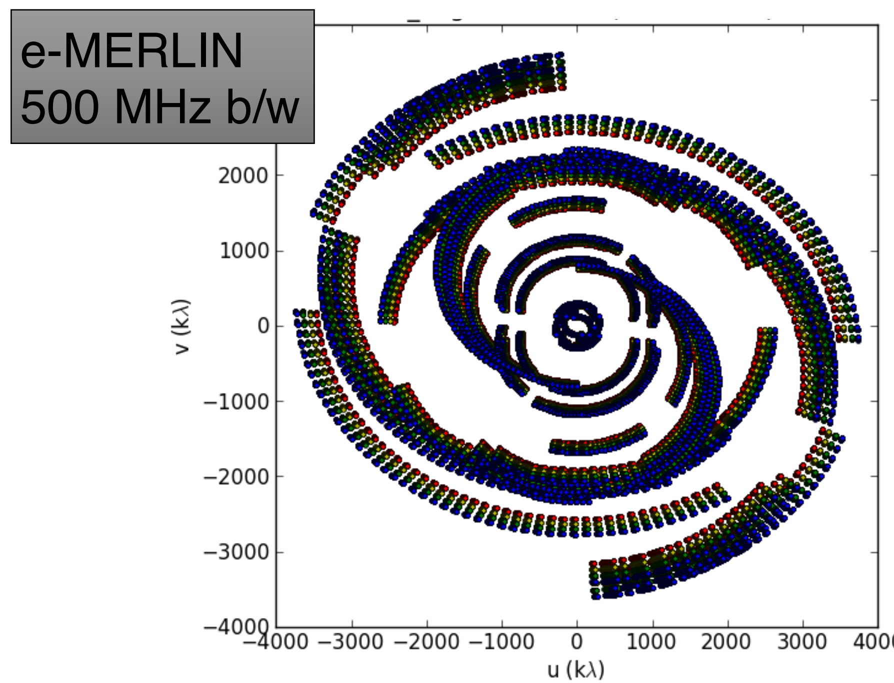
Solution: combine arrays?



Improving image fidelity

- Add more antennas?
  - Simulations tutorial on adding in AVN to VLBI
- Increase bandwidth?
  - Optical fibres and new correlators make possible





Radio interferometry +ives & -ives

- Highest resolution from Earth's surface
  - Spatial and spectral
- Observe day and night, rain or shine
  - (at least some radio wavelengths)
- Very small antenna position errors, long baselines
  - Very accurate astrometry
    - Using inertial frame of very distant quasars
- Gaps between antennas
  - Some scales are just not sampled
  - Large-scale smooth emission is invisible in images
    - (or, worse, such bright sources add noise!)
- Image reconstruction involves interpolation, deconvolution – complicated!
- Data sets can be huge (Tb or even more for SKA)